Patterns in Calabi–Yau Threefolds

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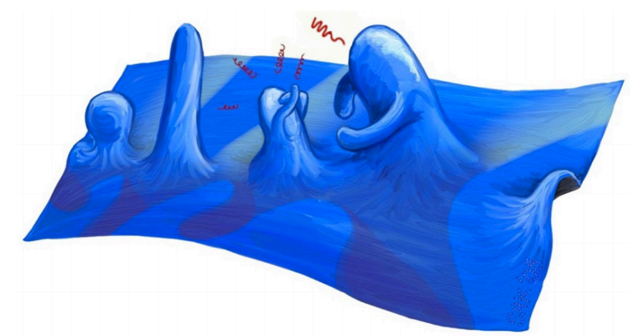
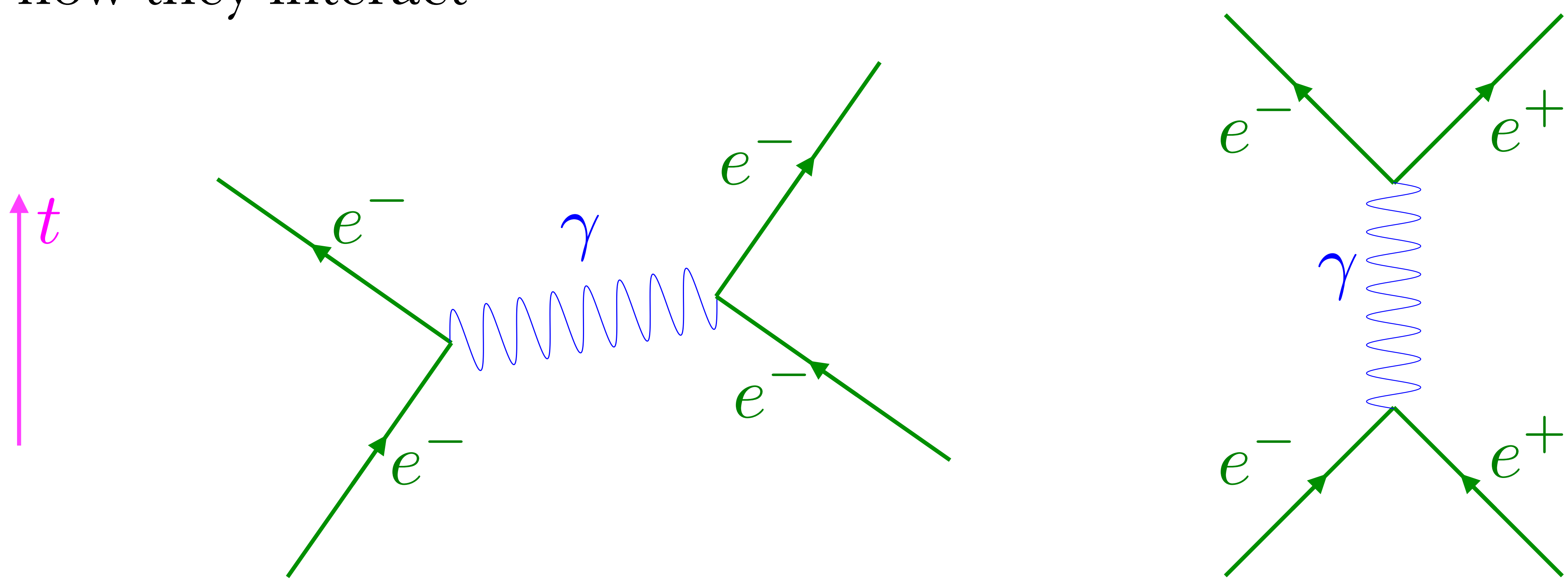
"Nonlinear Algebra in Applications" Workshop
Institute for Computational and Experimental Research in Mathematics
Brown University
12 November 2018

Quantum Field Theory

- A field has a value at every point in spacetime
- Particles are local excitations of these fields



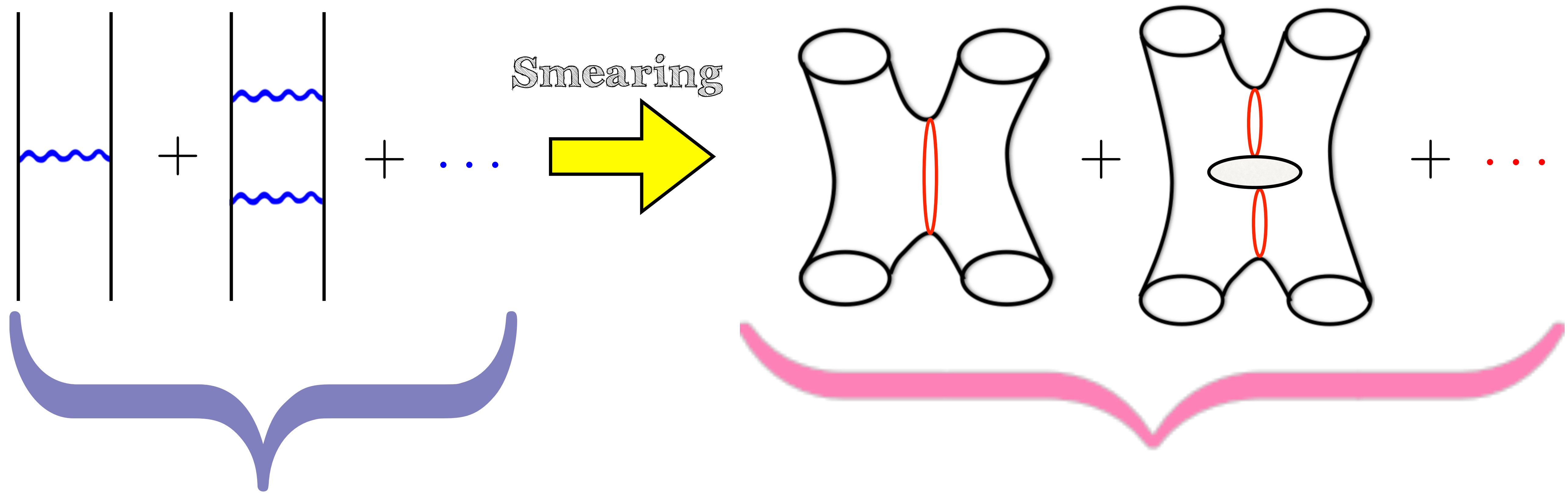
- To define a quantum field theory, we must specify the fields and how they interact



- Electrons and positrons interact by exchanging photons, for example

String Theory

Gravity as a QFT | **Gravity from String Theory**



These are infinite | These are finite

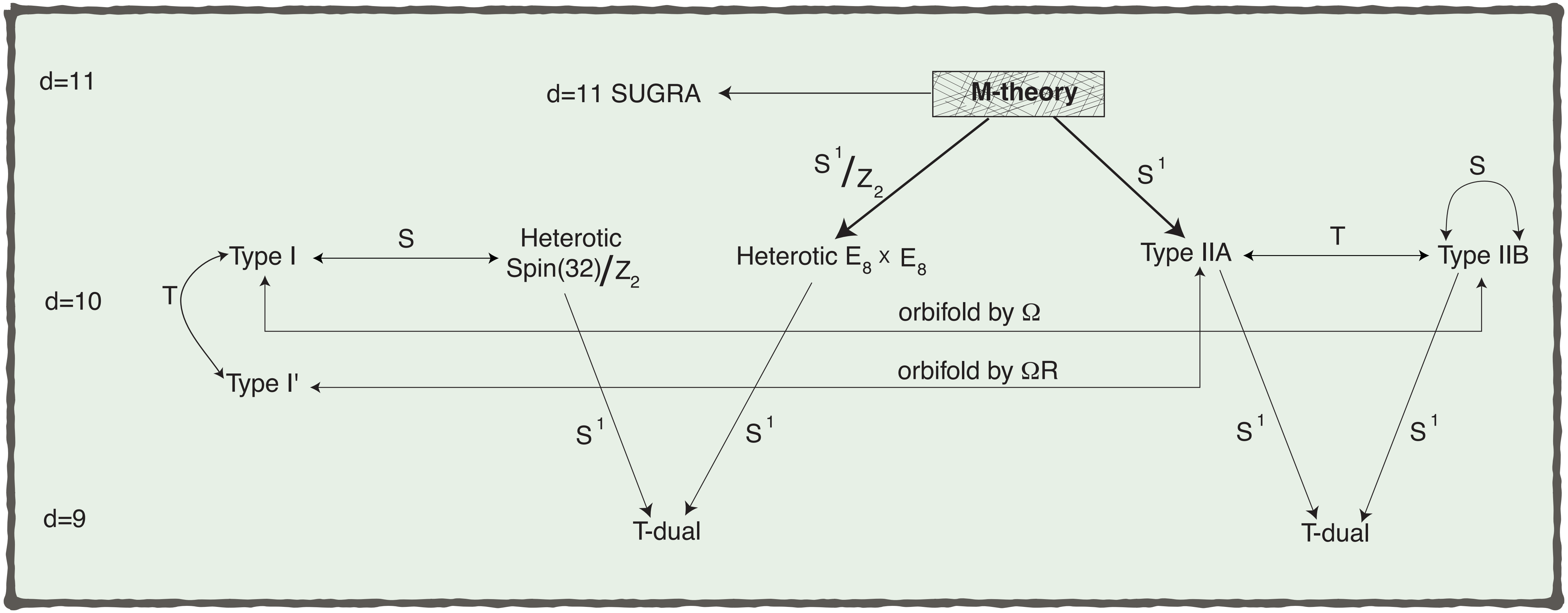
These are four dimensional | These are ten dimensional

[To prove the consistency of string theory we use the remarkable fact that $\sum_{n=1}^{\infty} n \longrightarrow -\frac{1}{12}$]

- $X^\mu : \Sigma \to \mathcal{M}$

  Sigma model on the string worldsheet gives general relativity

String Theory



- String theory is in fact a web of interconnected theories in ten (or eleven or twelve) dimensions

- How do we proceed?

The Forces of Nature

- Gravitational interactions described by Einstein

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu}$$

Newton, *Philosophiæ Naturalis Principia Mathematica* (1687)
Einstein, "On the General Theory of Relativity" (1915)

- Electromagnetism

Maxwell, Treatise on Electricity and Magnetism (1873)

- Weak force

Fermi (1933), Abdus-Salam, Glashow, Weinberg (1968)

- Mass mechanism

Brout, Englert; Higgs; Guralnik, Hagen, Kibble (1964)

- Strong force (quantum chromodynamics)

Yukawa (1935), Gell-Mann, Zweig (1961), Gross, Wilczek, Politzer (1973)

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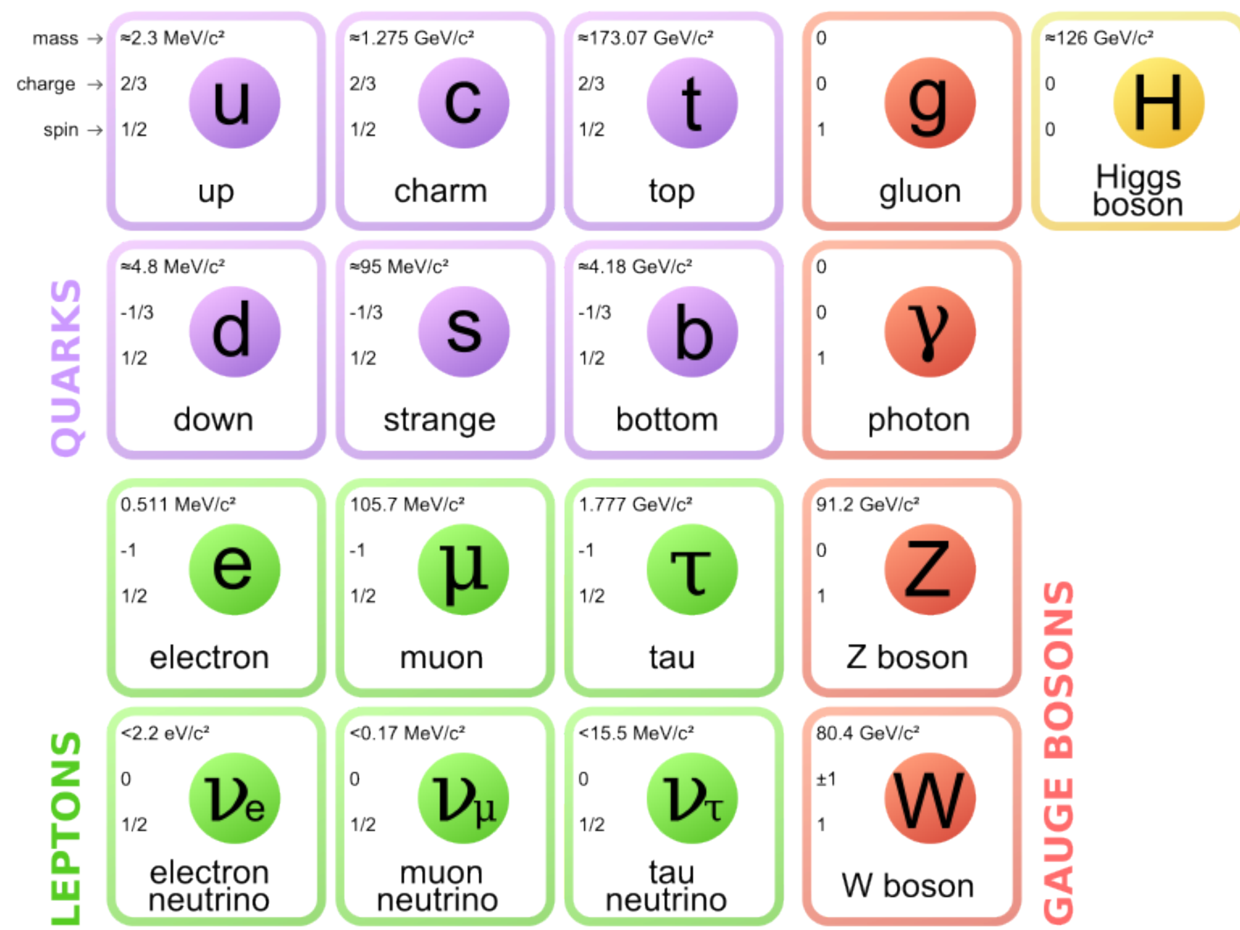
- Standard Model



$$\alpha_{\text{exp}}^{-1} = 137.035999139(31)$$

$$\alpha_{\text{th}}^{-1} = 137.035999173(35)$$

The Forces of Nature

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- Non-gravitational interactions are not encoded as geometry

Theorem [Coleman–Mandula]: symmetry group in 4 dimensions is Poincaré x internal

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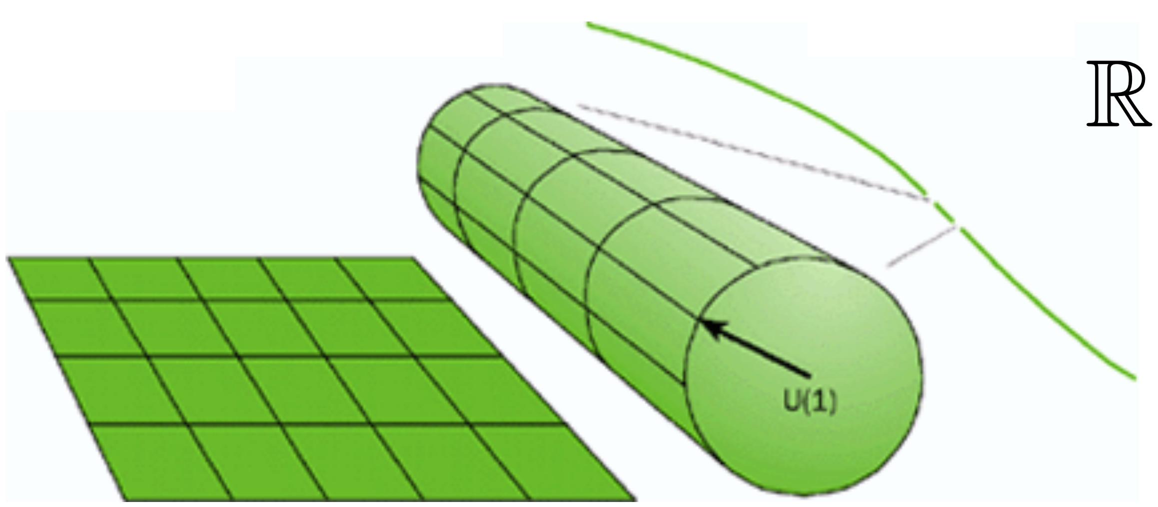
Theorem [Coleman–Mandula]: symmetry group in 4 dimensions is Poincaré x internal

- Clever loophole: internal symmetries may arise from higher dimensional geometry



$$\mathbb{R}^{1,3} \times S^1$$

Kaluza–Klein: 5d Einstein equations give 4d Einstein + Maxwell equations

Geometric Engineering

- Higher dimensional objects in string theory (branes) on which QFTs live

- Ten dimensional theory is consistent

- Ansatz for the geometry is $\mathcal{M}_{10} = \mathbb{R}^{1,3} \times \mathrm{CY}_3$

- Properties of Calabi–Yau determine physics in four dimensions

Example: $N_g = \frac{1}{2}|\chi|$ in simplest heterotic compactification models

Candelas, Horowitz, Strominger, Witten (1985)
Greene, Kirklin, Miron, Ross (1986)

Geometric Engineering

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$$\mathrm{dS}_4$$

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The Real World

- String theory supplies a framework for quantum gravity

- We are beginning to understand black holes and holography

- String theory is also an organizing principle for mathematics

- Finding our universe among the myriad of possible consistent realizations of a four dimensional low-energy limit of string theory is the **vacuum selection problem**

- Most vacua are *false* in that they do not resemble Nature at all

- Among the landscape of possibilities, we do not have even one solution that reproduces all the particle physics and cosmology we know

The Unreal World

- The objective is to obtain the real world from a string compactification

- We would happily settle for a modestly unreal world

$\mathcal{N}=1$ supersymmetry in 4 dimensions
$G = SU(3)_C \times SU(2)_L \times U(1)_Y$
Matter in chiral representations of G : $(\mathbf{3},\mathbf{2})_{\frac{1}{6}}, (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}}, (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}}, (\mathbf{1},\mathbf{2})_{\pm\frac{1}{2}}, (\mathbf{1},\mathbf{1})_1, (\mathbf{1},\mathbf{1})_0$
Superpotential $W \supset \lambda^{ij}\phi\overline{\psi}^i_L\psi^j_R$
Three copies of matter such that λ^{ij} not identical
Consistent with cosmology

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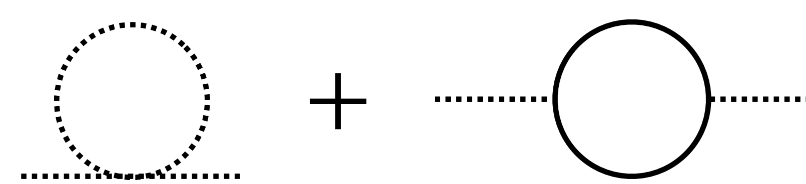
$$\boxed{\mathcal{N} = 1 \text{ supersymmetry in 4 dimensions}}$$

No experimental evidence so far!

$$Q|\lambda\rangle \sim |\lambda \pm \frac{1}{2}\rangle$$

$$|\text{boson}\rangle \longleftrightarrow |\text{fermion}\rangle$$



$$m_{\rm H} \ll m_{\rm Pl}$$

The Unreal World

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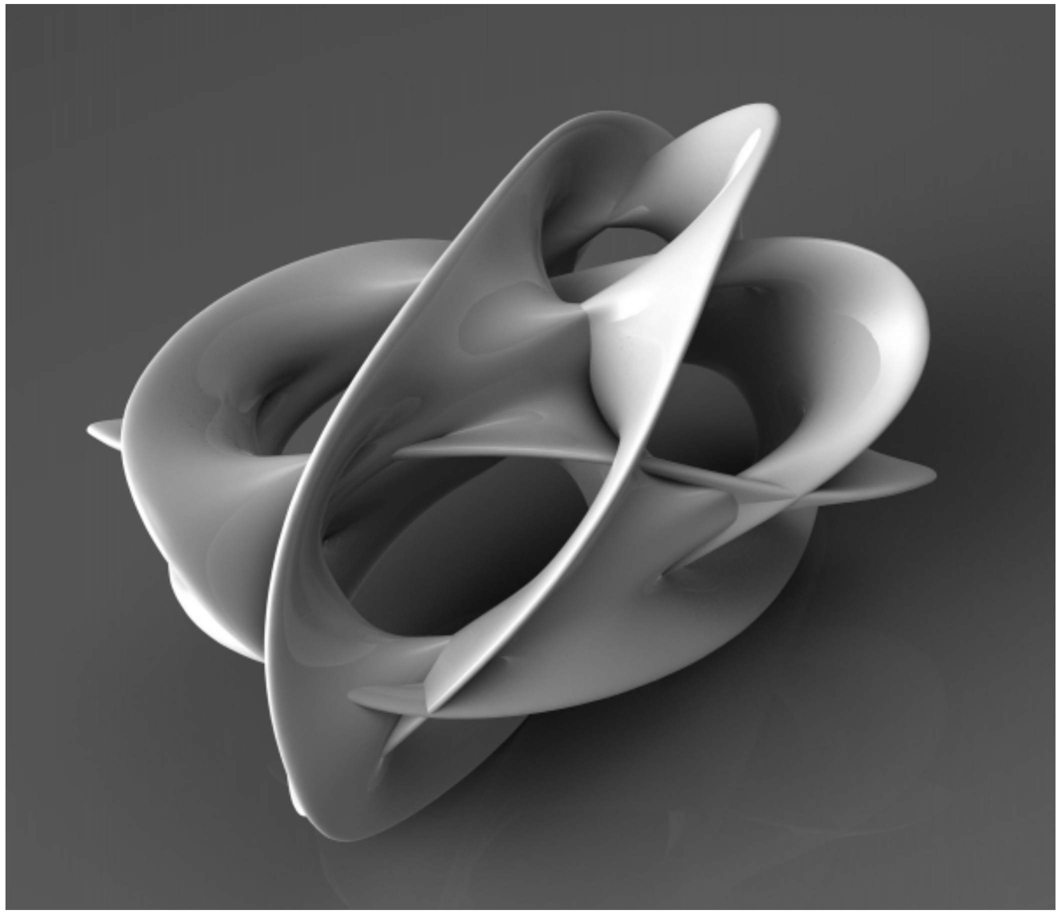
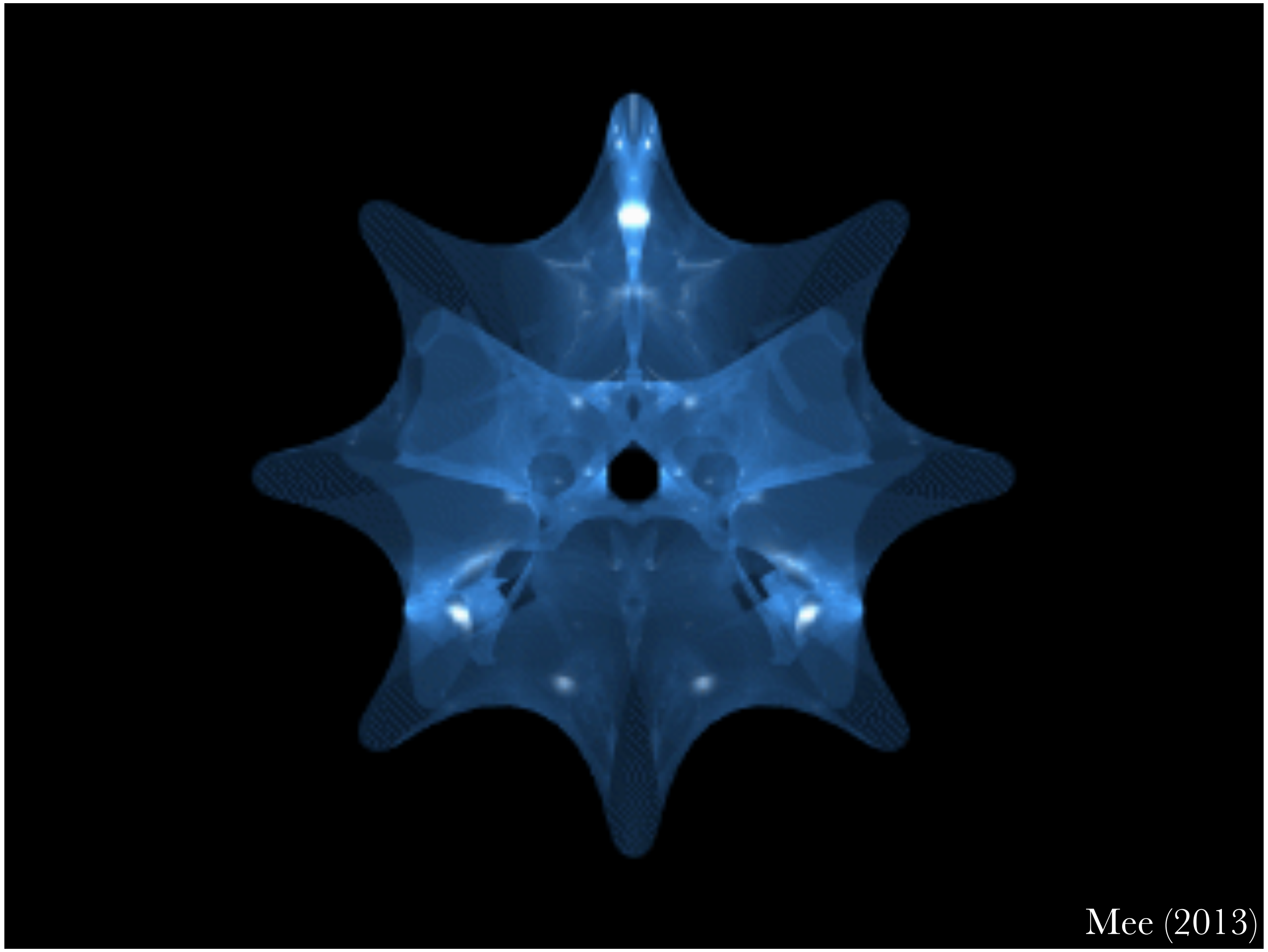
$$\boxed{\mathcal{N} = 1 \text{ supersymmetry in 4 dimensions}}$$

Because it is Ricci flat, the Calabi–Yau geometry ensures 4d supersymmetry

Use topological and geometric features of the Calabi–Yau to recover aspects of the real world

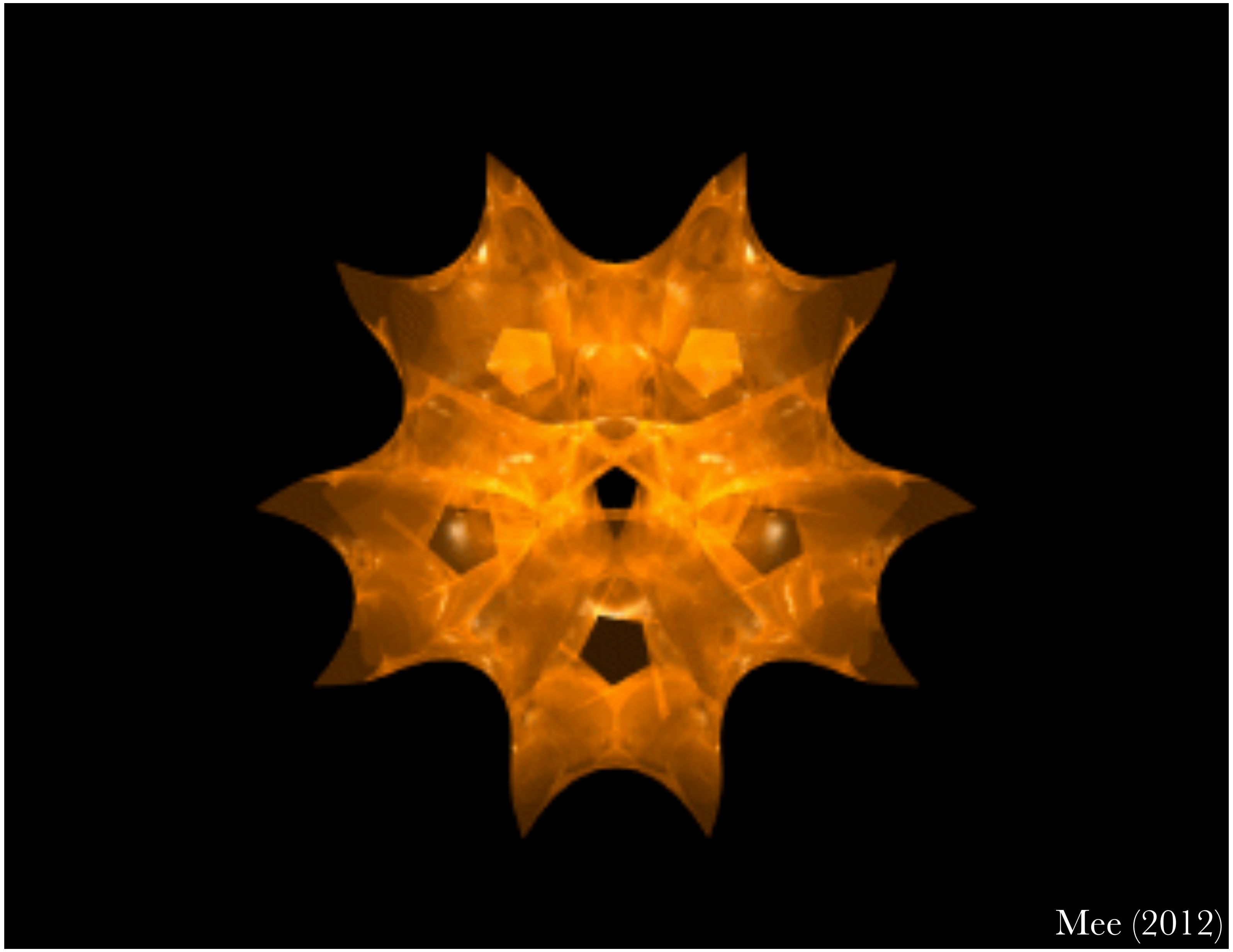


Calabi–Yau



$w^4 + x^4 + y^4 + z^4 = 0 \subset \mathbb{P}^3$

Calabi–Yau



$u^5 + v^5 + x^5 + y^5 + z^5 = 0 \subset \mathbb{P}^4$

There is a nowhere vanishing holomorphic n-form

The canonical bundle is trivial

There is a Kähler metric with global holonomy in $SU(n)$

Outline

- Introduction and motivation

- Patterns in distribution of Hodge numbers of toric Calabi–Yau threefolds

- Machine learning complete intersection Calabi–Yau threefolds (CICYs)

  — Hodge numbers

  — Favorability

  — Discrete symmetries

- Summary and prospects

Toric Varieties

- Consider: $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1 \subset \mathbb{C}^3$
- Identify $(z_1, z_2, z_3) \sim e^{i\phi}(z_1, z_2, z_3)$ to define $\mathbb{P}^2 \subset \mathbb{C}^3$
- Define $(x, y, z) = (|z_1|^2, |z_2|^2, |z_3|^2)$
- The original geometry is $x + y + z = 1$ or $z = 1 - x - y$

  This is a triangle $\mathcal{B}$
- Use $U(1)$ to choose the phase of z_3
- The phases of z_1 , z_2 define an algebraic torus over the base
- Open dense subset; action of torus on itself extends over the variety

From Polytopes to Geometries

- Formally, a **reflexive polytope** is defined as follows:

  The (possibly singular) toric variety A_{n+1} is specified by an integer polytope Δ in $\mathbb{R}^{n+1}$, which is a collection of vertices (dimension 0) each of which is an $(n+1)$-vector with integer entries, such that each pair of neighboring vertices defines an edge (dimension 1), each pair of edges defines a face (dimension 2), etc., all the way up to a facet (dimension n). The polytope is then the convex body in $\mathbb{R}^{n+1}$ enclosed by these facets. We will always include the origin as being contained in Δ. Using the usual dot product $\langle\ ,\ \rangle$ inherited from $\mathbb{R}^{n+1}$, the dual polytope is defined by

$$\Delta^\circ := \left\{ v \in \mathbb{R}^{n+1} | \langle m, v \rangle \geq -1, \forall\ m \in \Delta \right\}\ .$$

  The polytope Δ is **reflexive** if all the vertices of Δ° are integer vectors.

- From this, we compute the **Calabi–Yau hypersurface**:

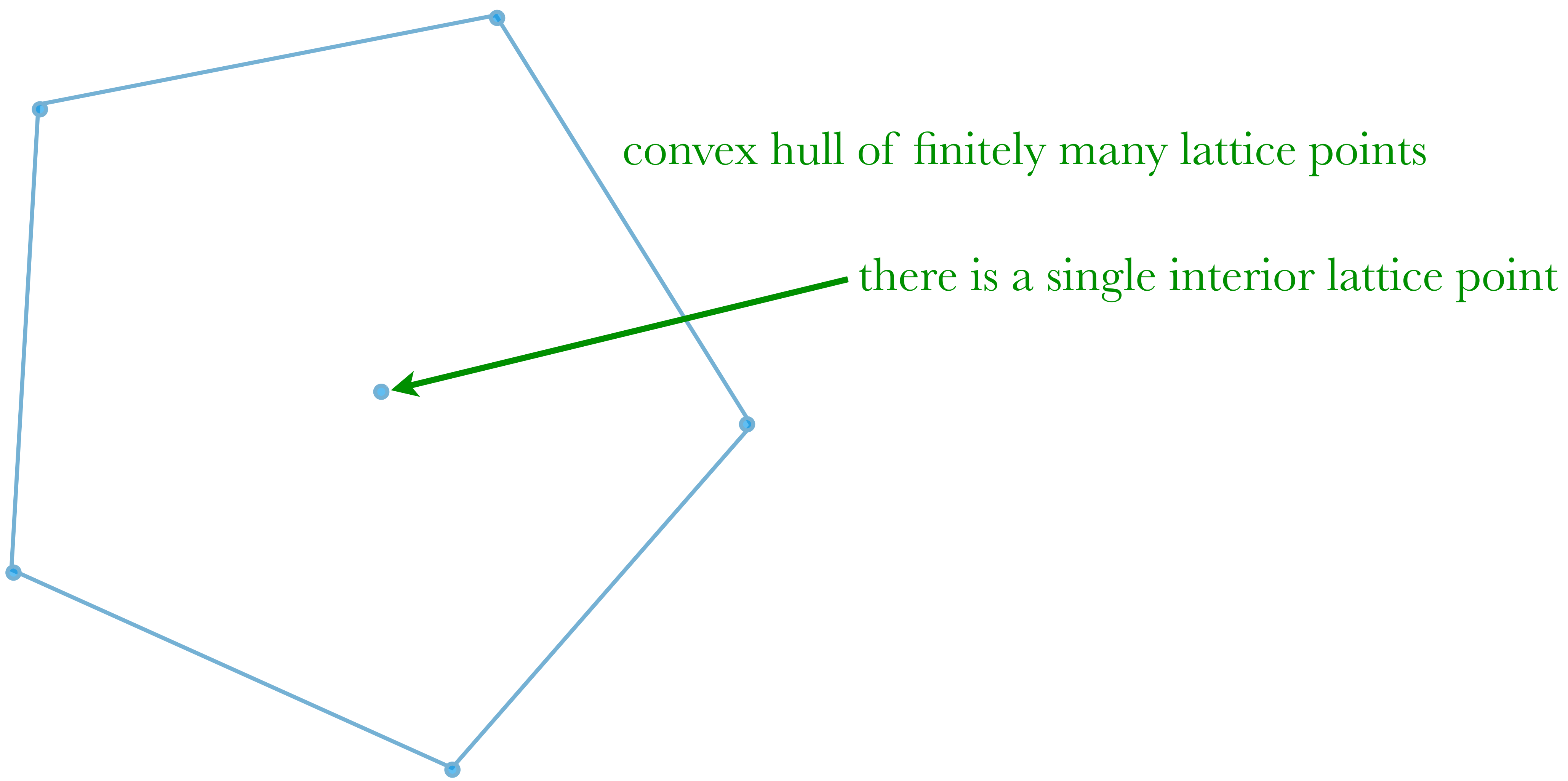
  We define the Calabi–Yau hypersurface X_n explicitly as the polynomial equation

$$\sum_{m \in \Delta} c_m \prod_{r=1}^{k} x_r^{\langle m, v_r \rangle + 1} = 0\ ,$$

  where $v_{r=1,\ldots,k}$ are the vertices of Δ° with k being the number of vertices of Δ° (or equivalently the number of facets of Δ), x_r are the coordinates of A_{n+1}, and c_m are numerical coefficients parameterizing the complex structure of X_n.

Reflexive Polytopes

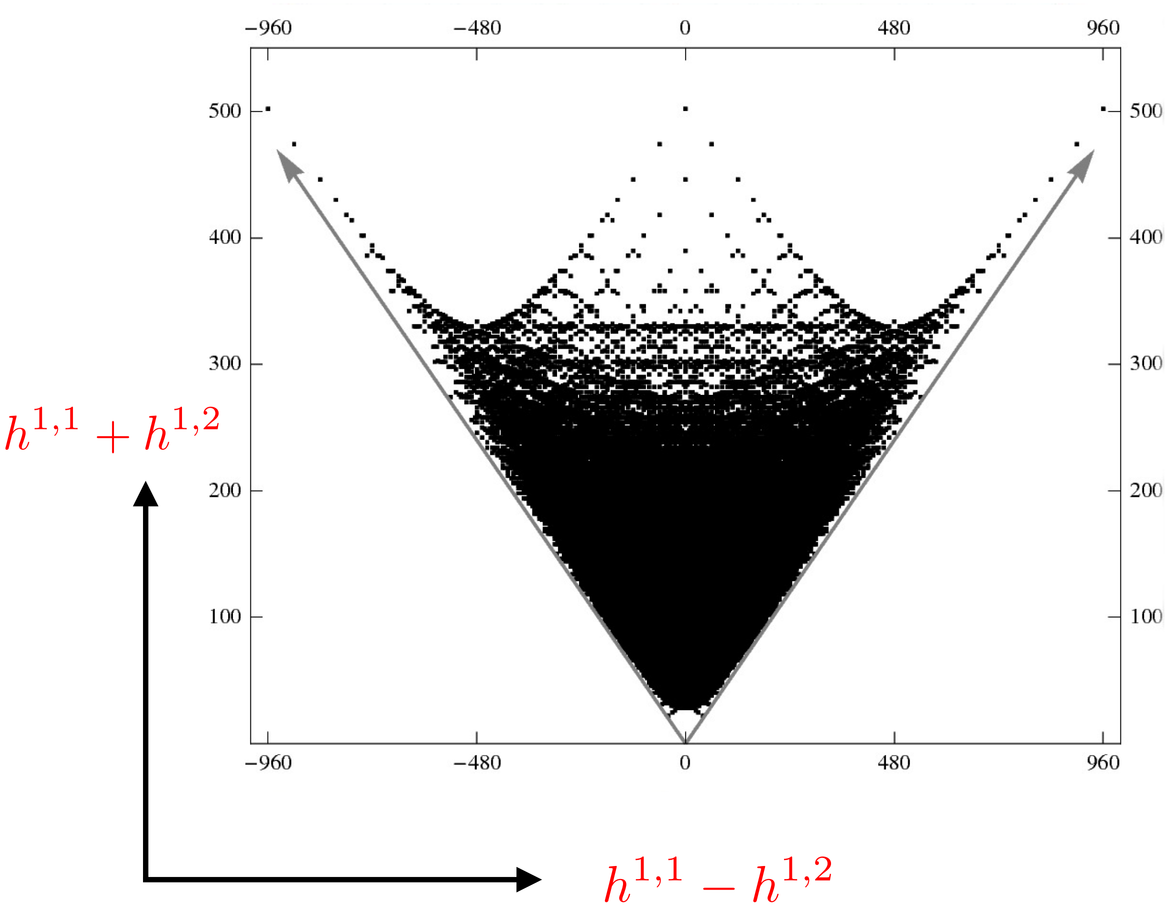
- Starting from a reflexive polytope, one can build a toric Calabi–Yau via methods of Batyrev, Borisov



Reflexive Polytopes Catalogued

- Starting from a reflexive polytope, one can build a toric Calabi–Yau via methods of Batyrev, Borisov

- Kreuzer–Skarke obtained 473,800,776 reflexive polytopes that yield toric Calabi–Yau threefolds with 30,108 unique pairs of Hodge numbers



- Distribution of polytopes exhibits mirror symmetry

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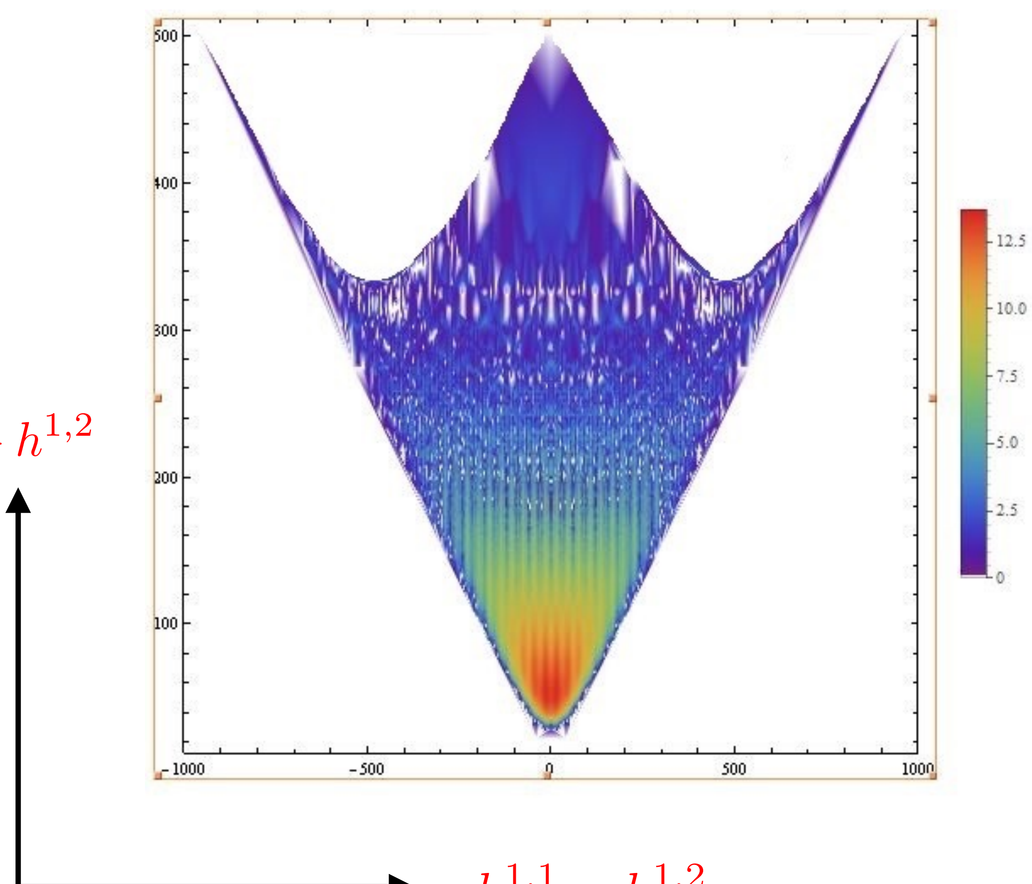
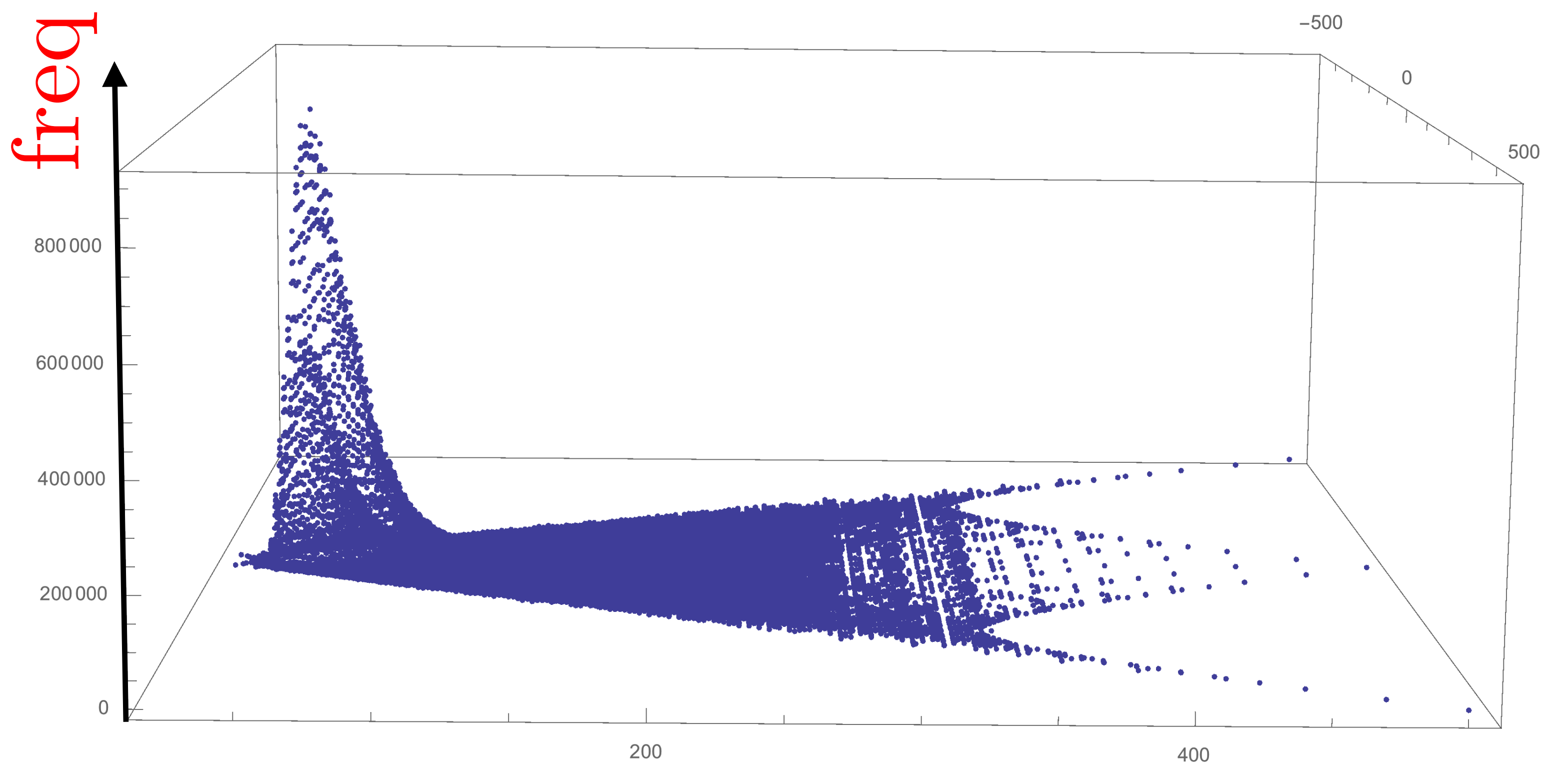
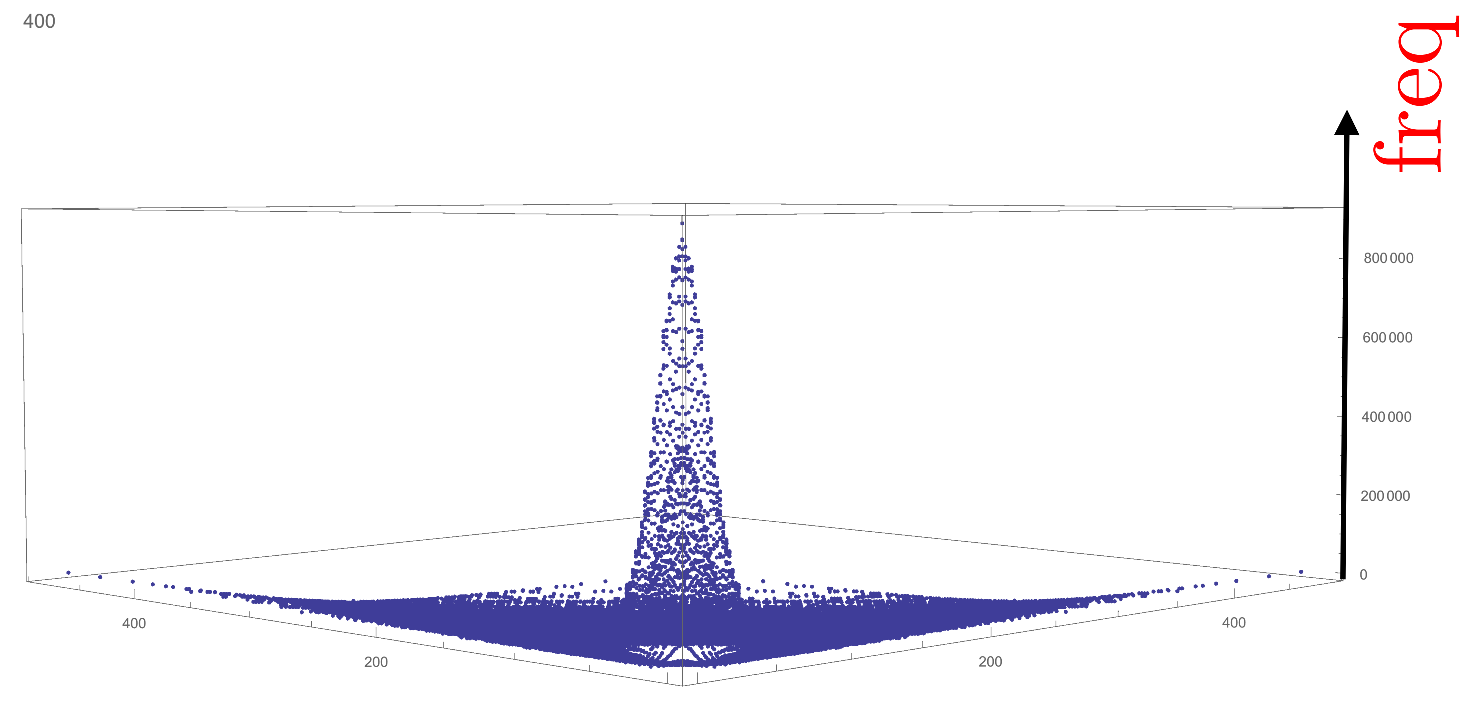


- Distribution of polytopes exhibits mirror symmetry
- The peak of the distribution is at $(h^{1,1}, h^{1,2}) = (27, 27)$

  There are 910,113 such polytopes
- Are there patterns in how the topological invariants are distributed?

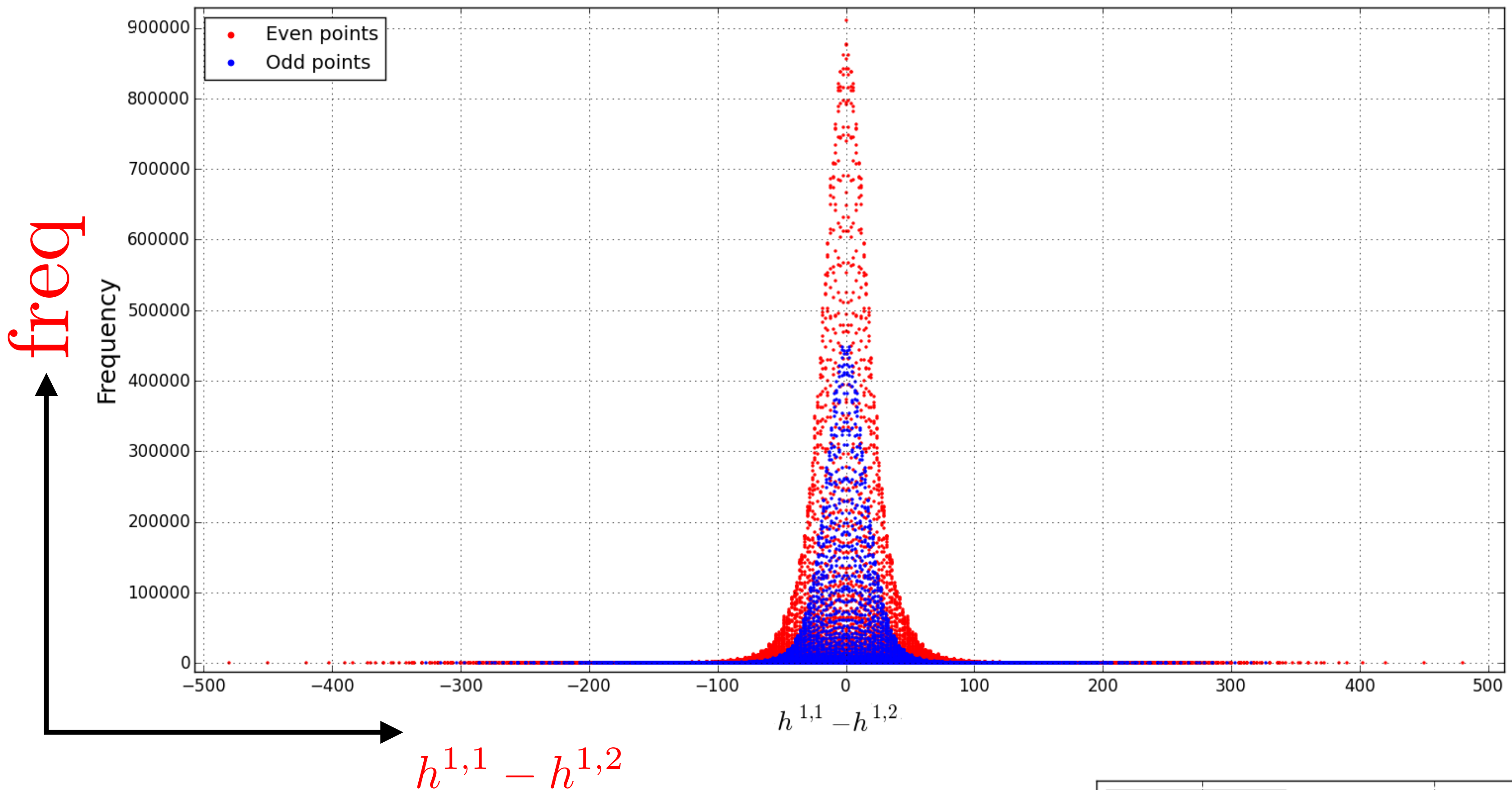
3d Plots of Polytope Data

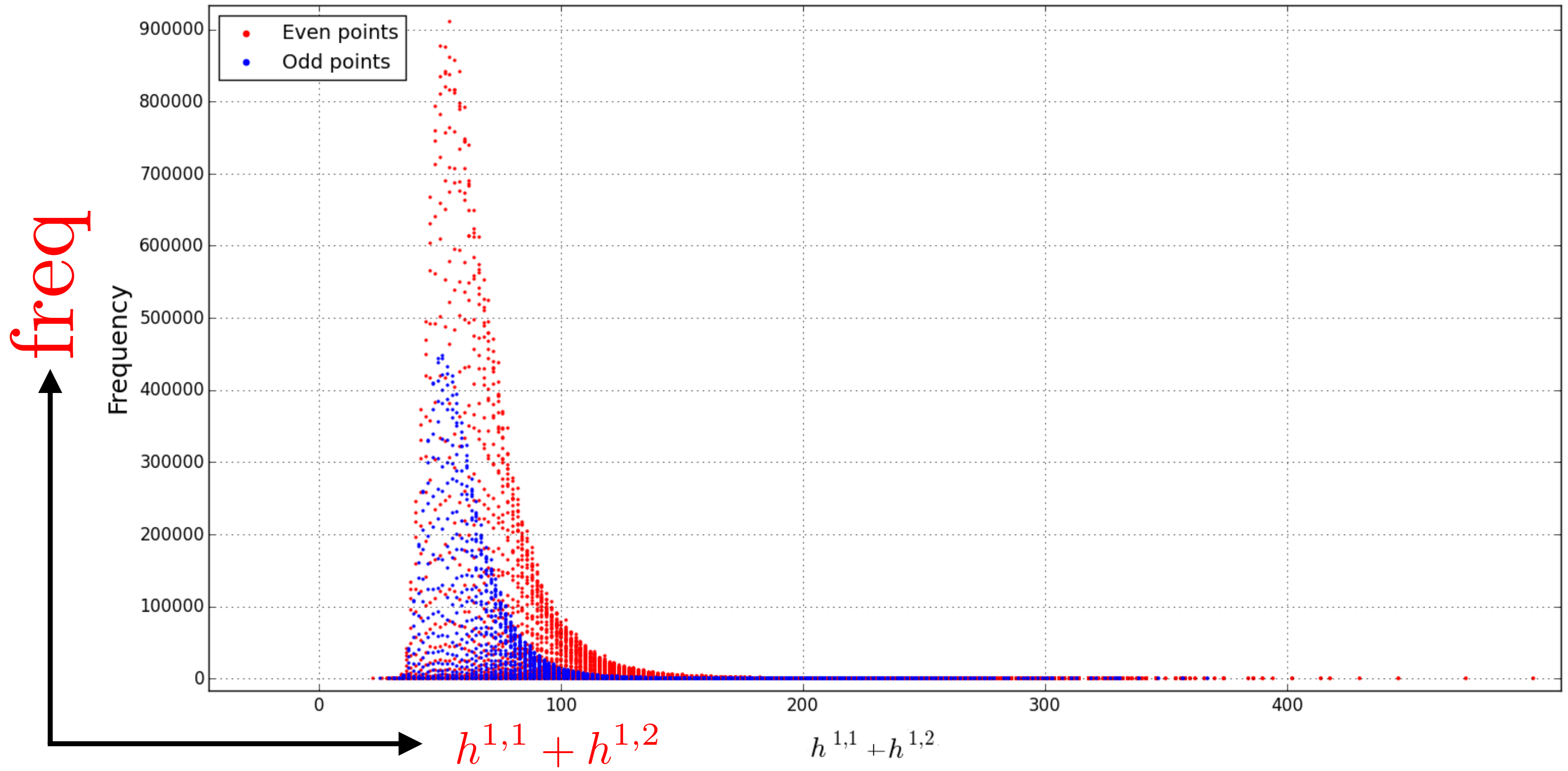




He, VJ, Pontiggia (2015)

Patterns in CY Distributions





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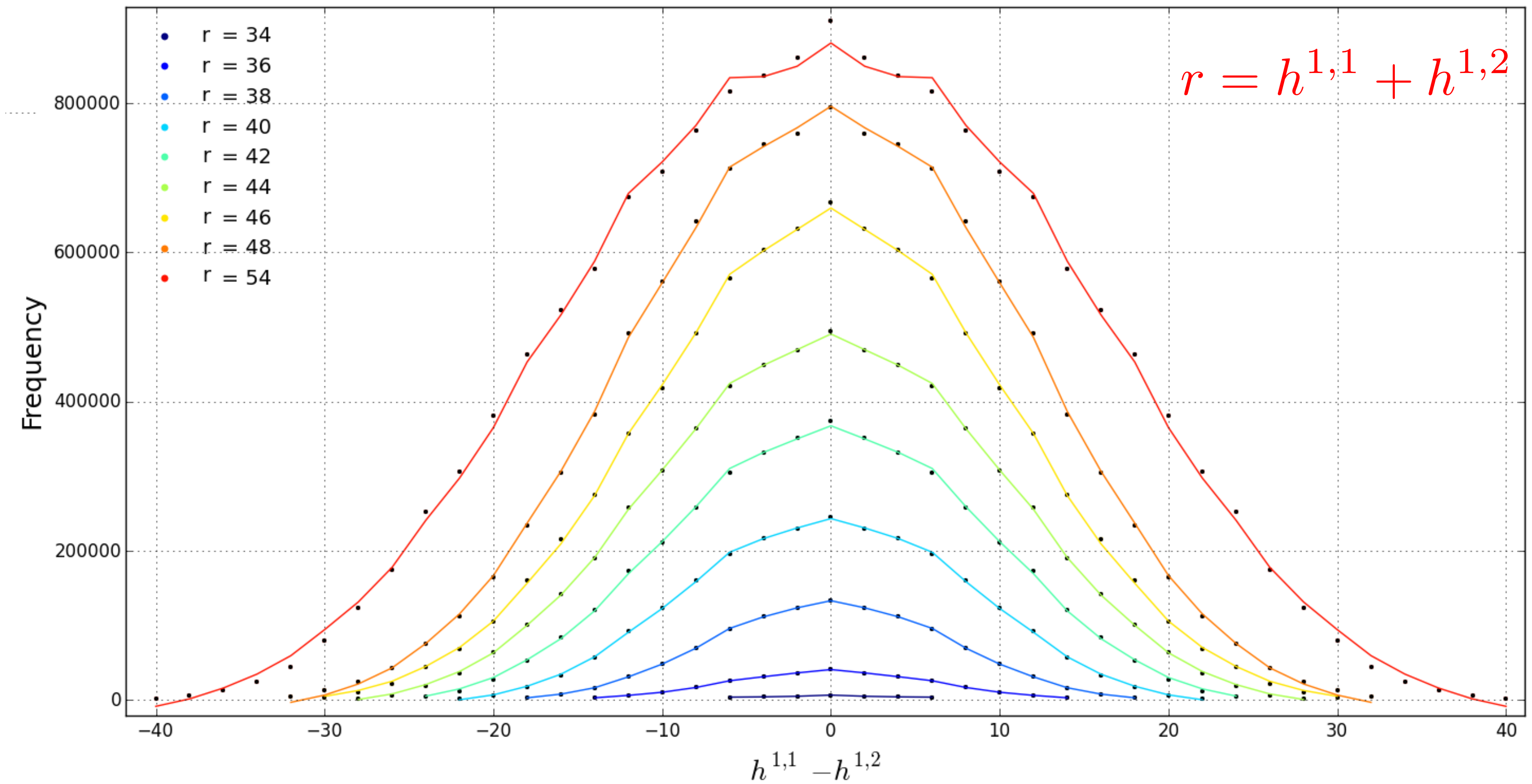


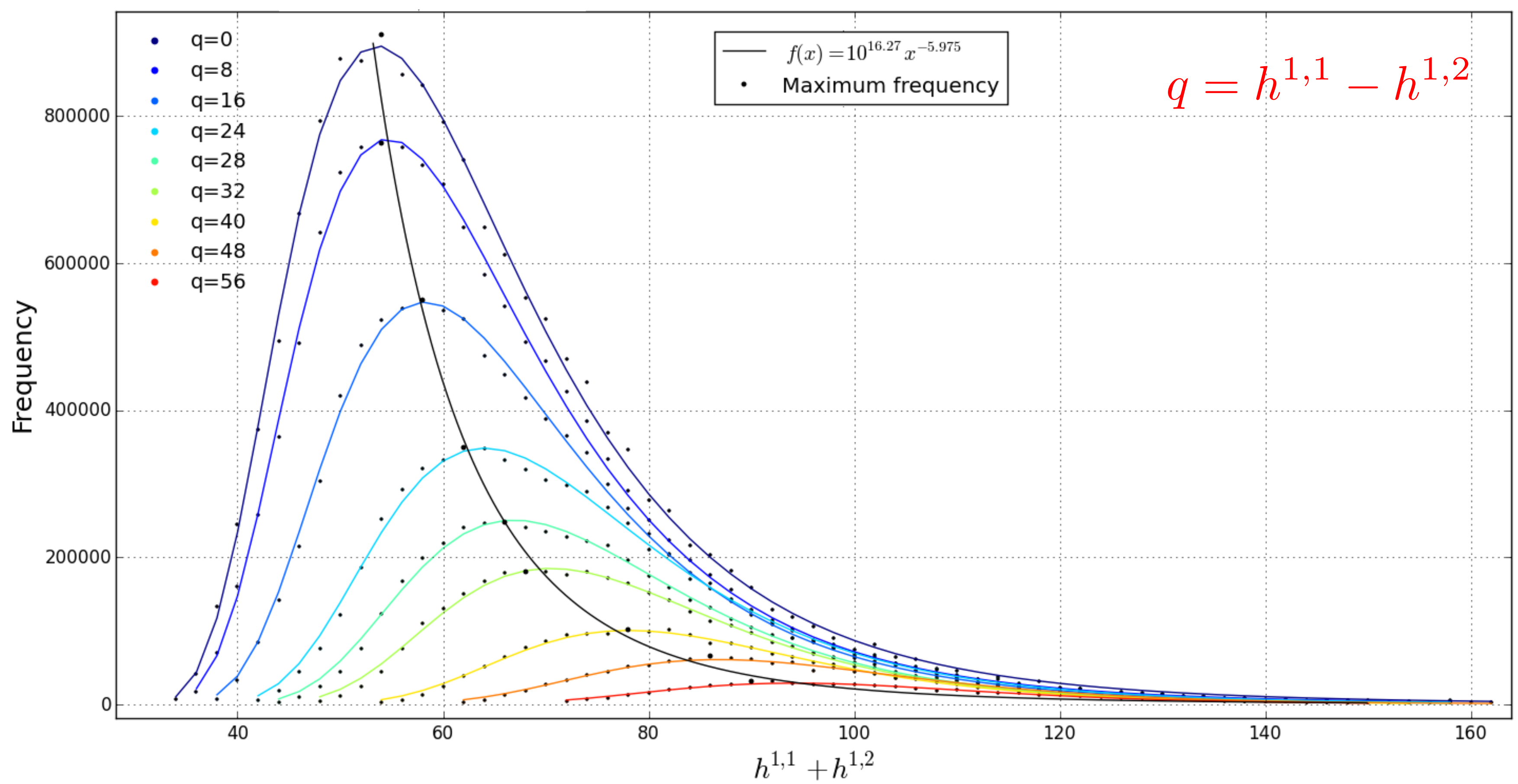
Pseudo-Voigt distribution

sum of Gaussian and Cauchy

$$(1-\alpha)\frac{A}{\sigma\sqrt{2\pi}}e^{-\frac{(x-\mu)^2}{2\sigma^2}} + \alpha\frac{A}{\pi}\left[\frac{\sigma^2}{(x-\mu)^2+\sigma^2}\right]$$

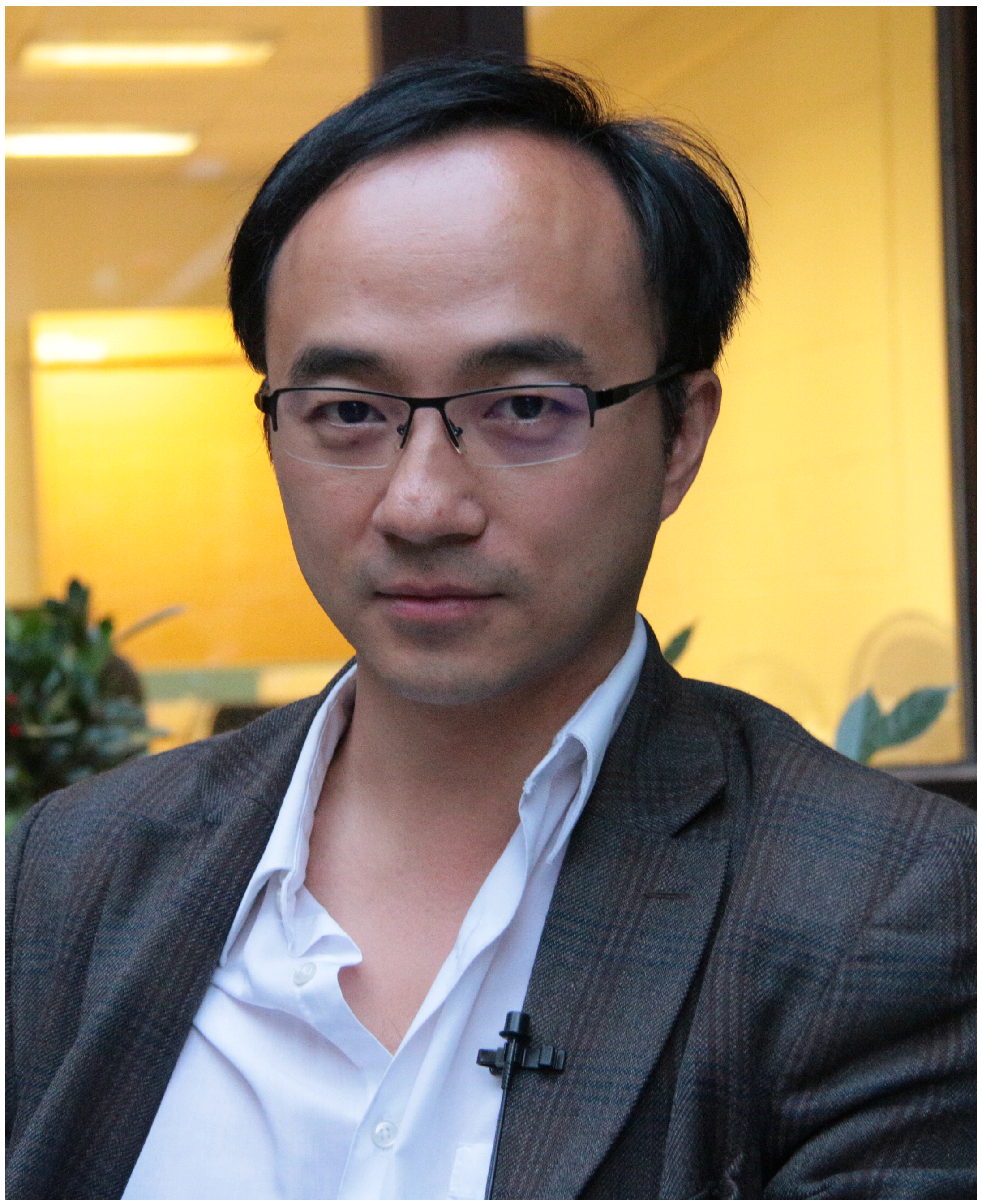
Planck distribution

$$\frac{A}{x^n}\frac{1}{e^{b/(x-c)}-1}$$

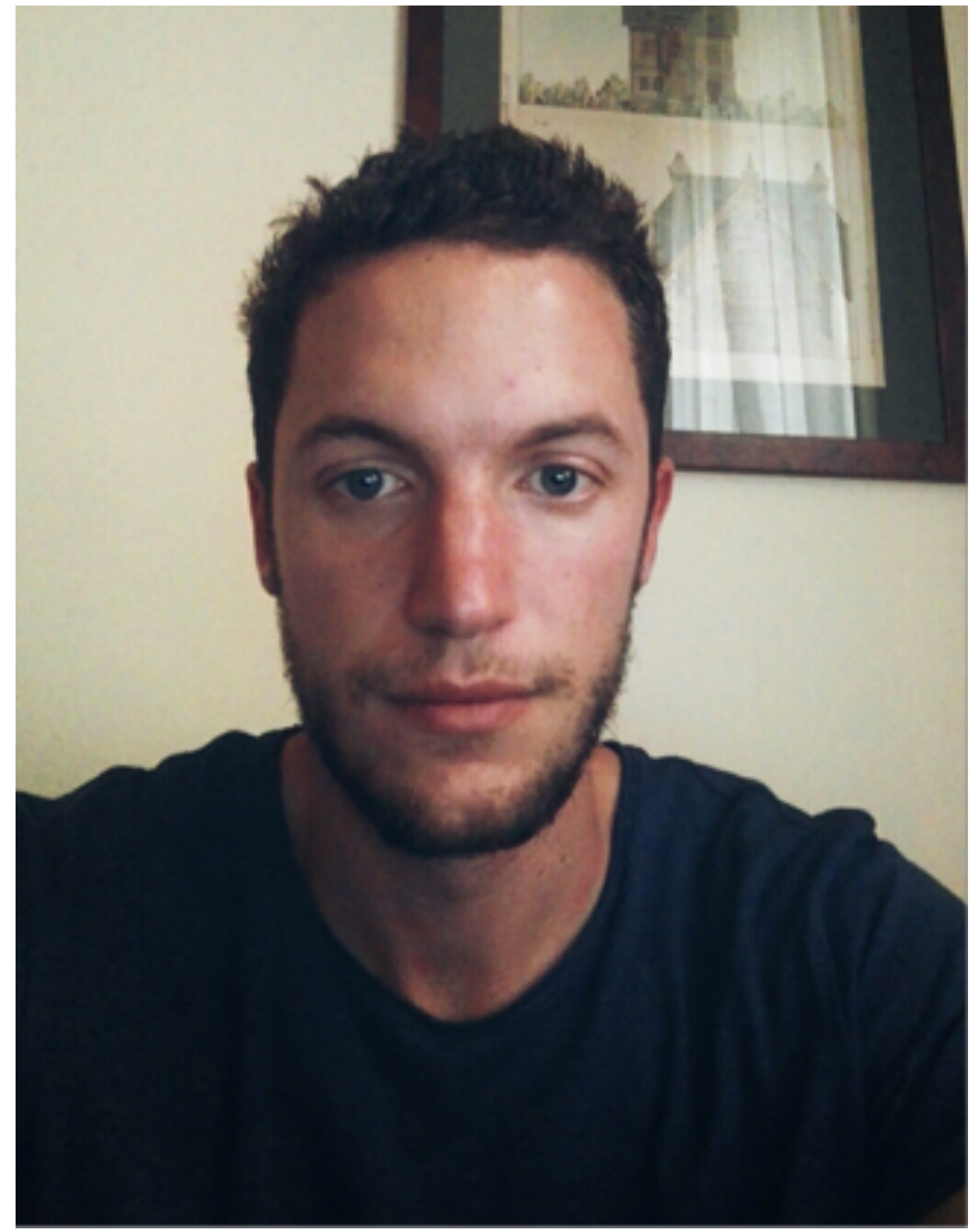


He, VJ, Pontiggia (2015)

Collaborators



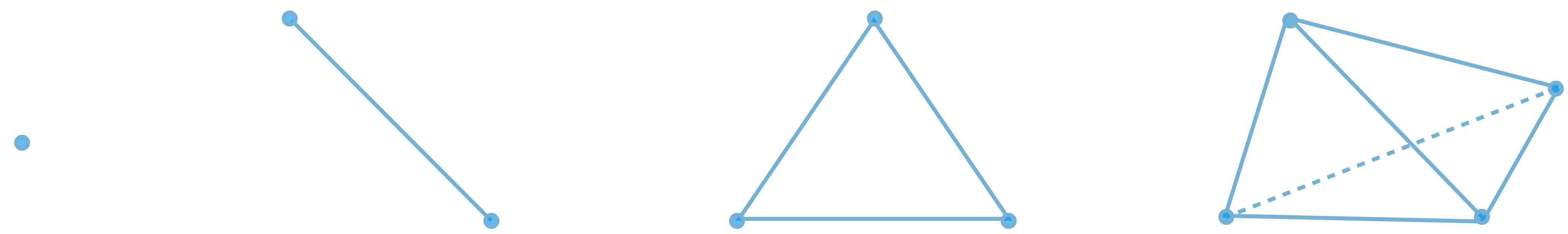
Yang-Hui He



Luca Pontiggia

arXiv:1512.01579

From Polytopes to Geometries



0-simplex 1-simplex 2-simplex 3-simplex ...

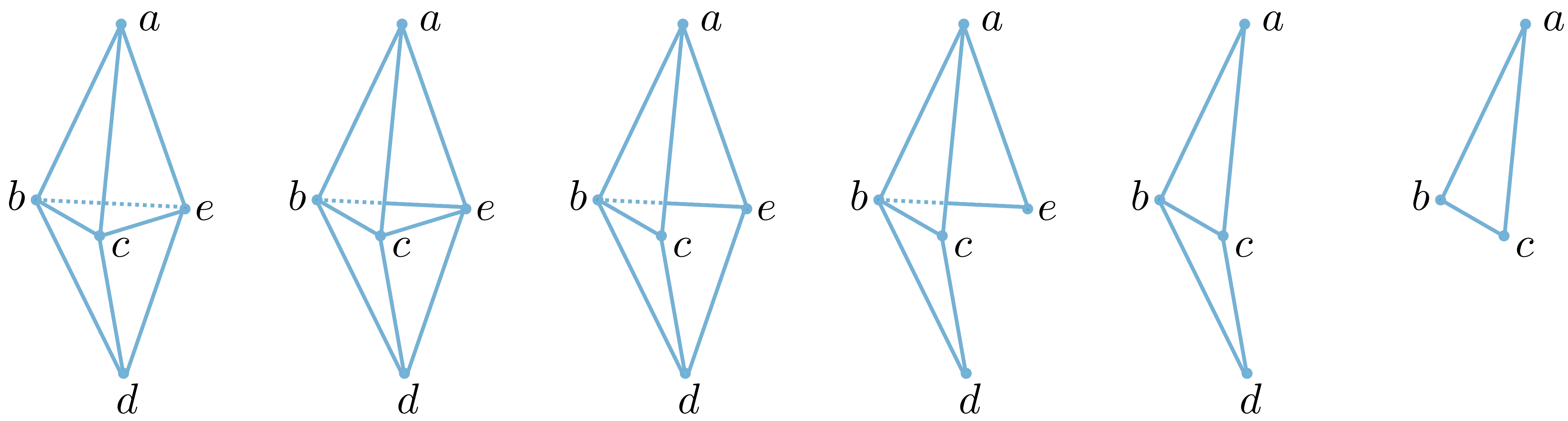
- A **triangulation** of $\mathcal{P}$ is a partition into simplices such that:

  the union of all simplices is $\mathcal{P}$

  the intersection of any pair is a (possibly empty) common face

- From triangulation, we construct the Stanley–Reisner ring
- Unique rings correspond to different Calabi–Yau geometries
- For each, we have topological data, intersection form, Kähler cone

Example: S^2



$I_\Delta = (ad, bce)$ minimal non-faces

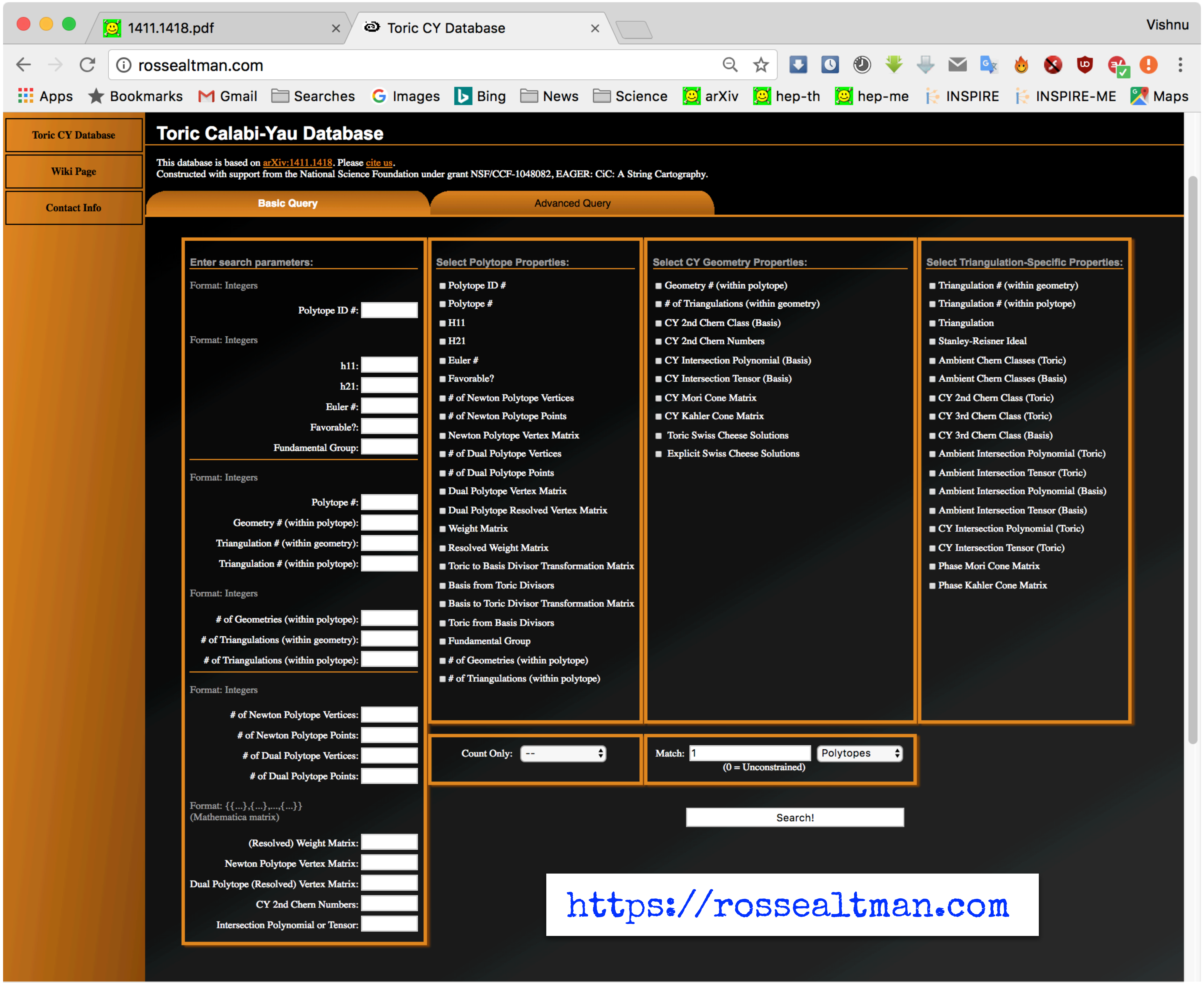
$\mathbb{K}_\Delta = \mathbb{K}[a, b, c, d, e]/I_\Delta$ Stanley–Reisner ring

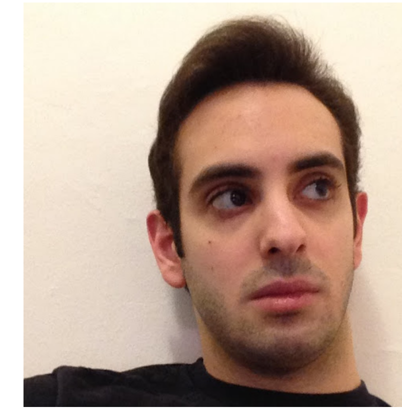
Homeomorphic to two-sphere

From Polytopes to Geometries

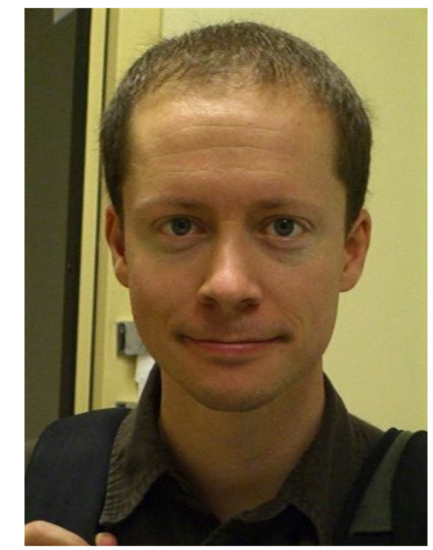
- Every triangulation of a reflexive polytope can yield a Calabi–Yau
- We do not know how many toric Calabi–Yau geometries there are
- Different triangulations of the same polytope are expected, in general, to give different Calabi–Yau manifolds
- In principle, triangulations of different polytopes can give the same Calabi–Yau manifold
- The Calabi–Yau inherits topological invariants from the polytope
- 16 polytopes in $\mathbb{R}^2$ give rise to elliptic curves (Calabi–Yau onefolds)
  4319 polytopes in $\mathbb{R}^3$ give rise to K3 (Calabi–Yau twofolds)
  473800776 polytopes in $\mathbb{R}^4$ give rise to at least 30108 Calabi–Yau threefolds

A Calabi–Yau Database





Ross Altman



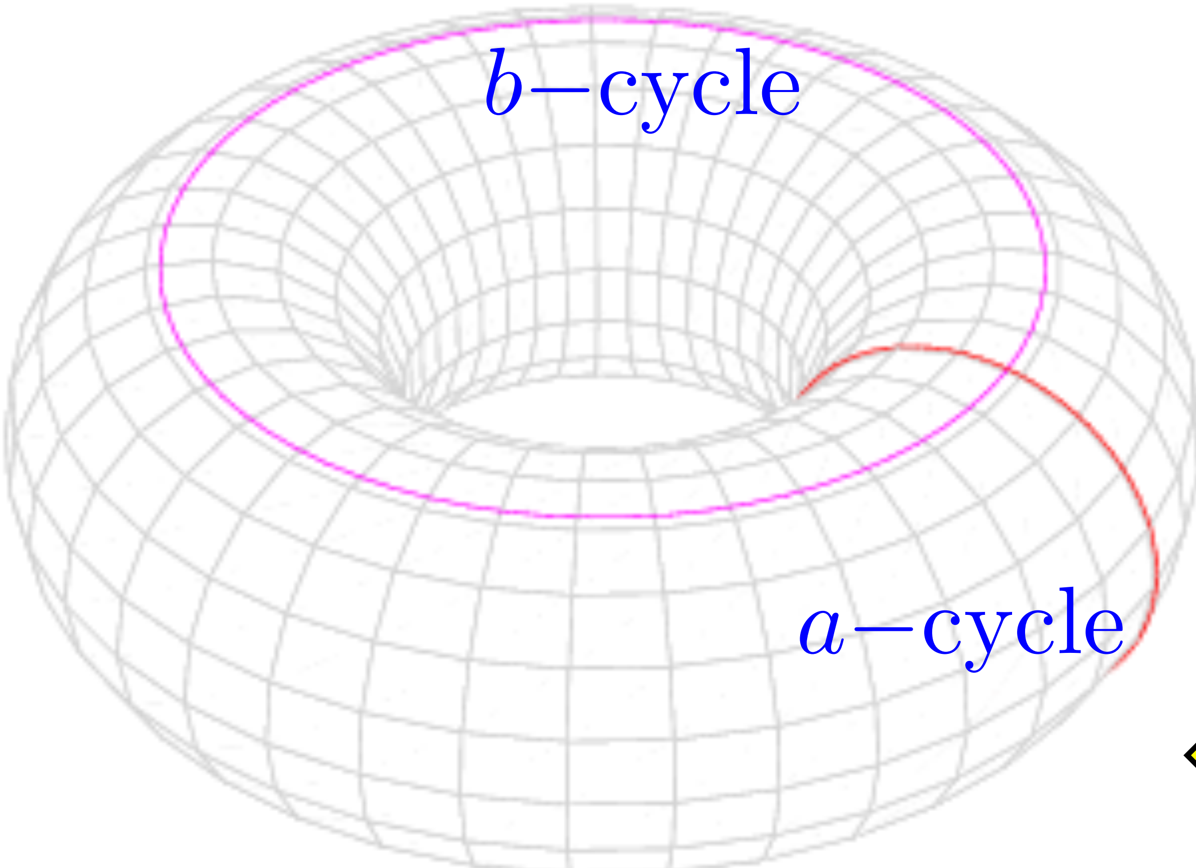
James Gray

Yang-Hui He

VJ

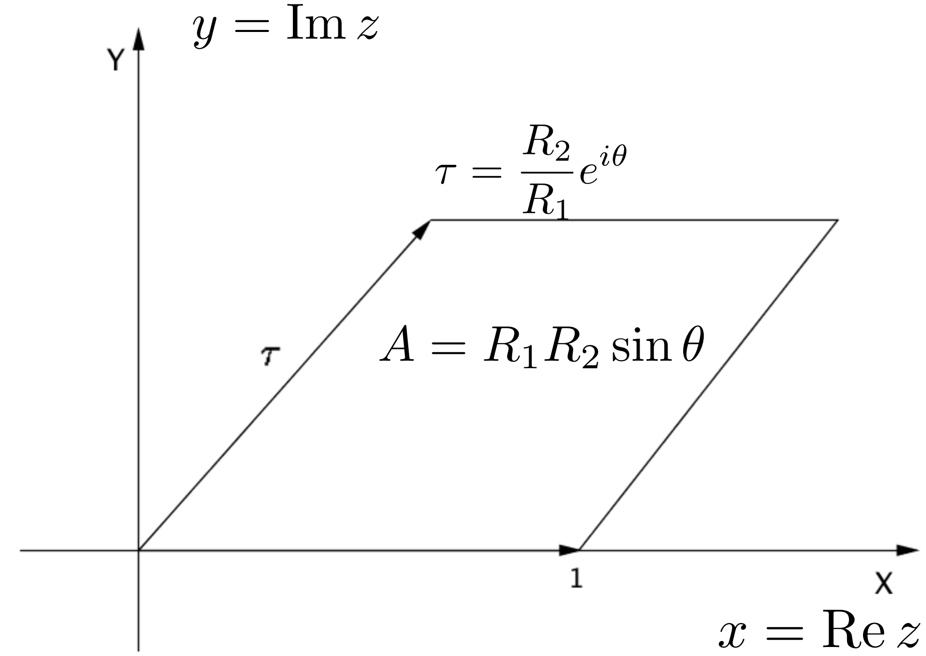
Brent Nelson

Torus



Flat, but has non-trivial homotopy

There are non-contractible cycles



Kähler parameter: area A **size**

complex structure parameter: τ **shape**

$$\mathrm{d}s^2 = R_1^2 dx^2 + R_2^2 dy^2 + 2R_1 R_2 \cos\theta dx\, dy$$

Moduli of CY$_3$

- Geometrical moduli enumerated by number of embedded two-spheres and three-spheres

$$
\begin{array}{ccccccccc}
 & & & 1 & & & & & b_0 \\
 & & 0 & & 0 & & & & b_1 \\
 & 0 & & h^{1,1} & & 0 & & & b_2 \\
1 & & h^{1,2} & & h^{2,1} & & 1 & & b_3 \\
 & 0 & & h^{2,2} & & 0 & & & b_4 \\
 & & 0 & & 0 & & & & b_5 \\
 & & & 1 & & & & & b_6
\end{array}
$$

$$h^{p,q} = \dim\ H^{p,q}$$

$$b_k = \dim\ H^k = \sum_{p+q=k} h^{p,q}$$

$h^{p,q} = h^{q,p}$ (complex conjugation)

$h^{p,q} = h^{n-p,n-q}$ (Poincaré duality)

$$\chi = \sum_{p,q} (-1)^{p+q} h^{p,q}$$

$h^{1,2} = \dfrac{b_3}{2} - 1$ **complex structure moduli**, counts the number of three-cycles

$h^{1,1} = b_2$ **Kähler moduli**, counts the number of two-cycles and four-cycles

$\chi = 2(h^{1,1} - h^{1,2})$ **Euler characteristic**, $N_g = \dfrac{1}{2}|\chi|$

- **Mirror symmetry** says that we can rotate the Hodge diamond by $\pi/2$ and get a new Calabi–Yau with $h^{1,1} \leftrightarrow h^{1,2}$

CICYs

- Zero locus of a set of homogeneous polynomials over combined set of coordinates of projective spaces

$$X = \begin{array}{c} \mathbb{P}^{n_1} \\ \vdots \\ \mathbb{P}^{n_m} \end{array} \left[\begin{array}{ccc} q_1^1 & \cdots & q_K^1 \\ \vdots & \ddots & \vdots \\ q_1^m & \cdots & q_K^m \end{array} \right]$$

configuration matrix

$$\sum_r n_r - K = 3$$

complete intersection threefold

$$\sum_a q_a^r = n_r + 1 \,, \qquad \forall\, r \in \{1, \ldots, m\}$$

$$c_1 = 0$$

- K equations of multi-degree $q_a^r \in \mathbb{Z}_{\geq 0}$ embedded in $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$

- **Example:** quintic $\mathbb{P}^4(5)$

$$4 - 1 = 3$$

$$5 = 4 + 1$$

- Other examples: $\mathbb{P}^5(3,3)$, $\mathbb{P}^5(4,2)$, $\mathbb{P}^6(3,2,2)$, $\mathbb{P}^7(2,2,2,2)$

CICYs

- Tian–Yau manifold: $\begin{array}{c} \mathbb{P}^3 \\ \mathbb{P}^3 \end{array} \begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 1 \end{pmatrix}$

  has $\chi = -18$

  freely acting $\mathbb{Z}_3$ quotient gives manifold with $\chi = -6$

  central to early string phenomenology

- Transpose is Schön's manifold, also Calabi–Yau

$$\begin{array}{c} \mathbb{P}^2 \\ \mathbb{P}^2 \\ \mathbb{P}^1 \end{array} \begin{pmatrix} 3 & 0 \\ 0 & 3 \\ 1 & 1 \end{pmatrix}_{\chi = 0}$$

- Can compute χ from configuration matrix

CICYs

- For threefolds, we have constraints on size of configuration matrix

$$K \leq N_1 + N_a + 3 \ , \ \ N_1 \leq 9 \ , \ \ N_a \leq 6$$

number of other projective space factors

number of $\mathbb{P}^1$s

- We have: 7890 configuration matrices

Candelas, He, Hübsch, Lutken, Lynker, Schimmrigk, Berglund (1986-1990)

1×1 to 12×15 with $q_a^r \in [0,5]$

266 distinct Hodge pairs $(h^{1,1}, h^{1,2}) = (1,65), \ldots, (19,19)$

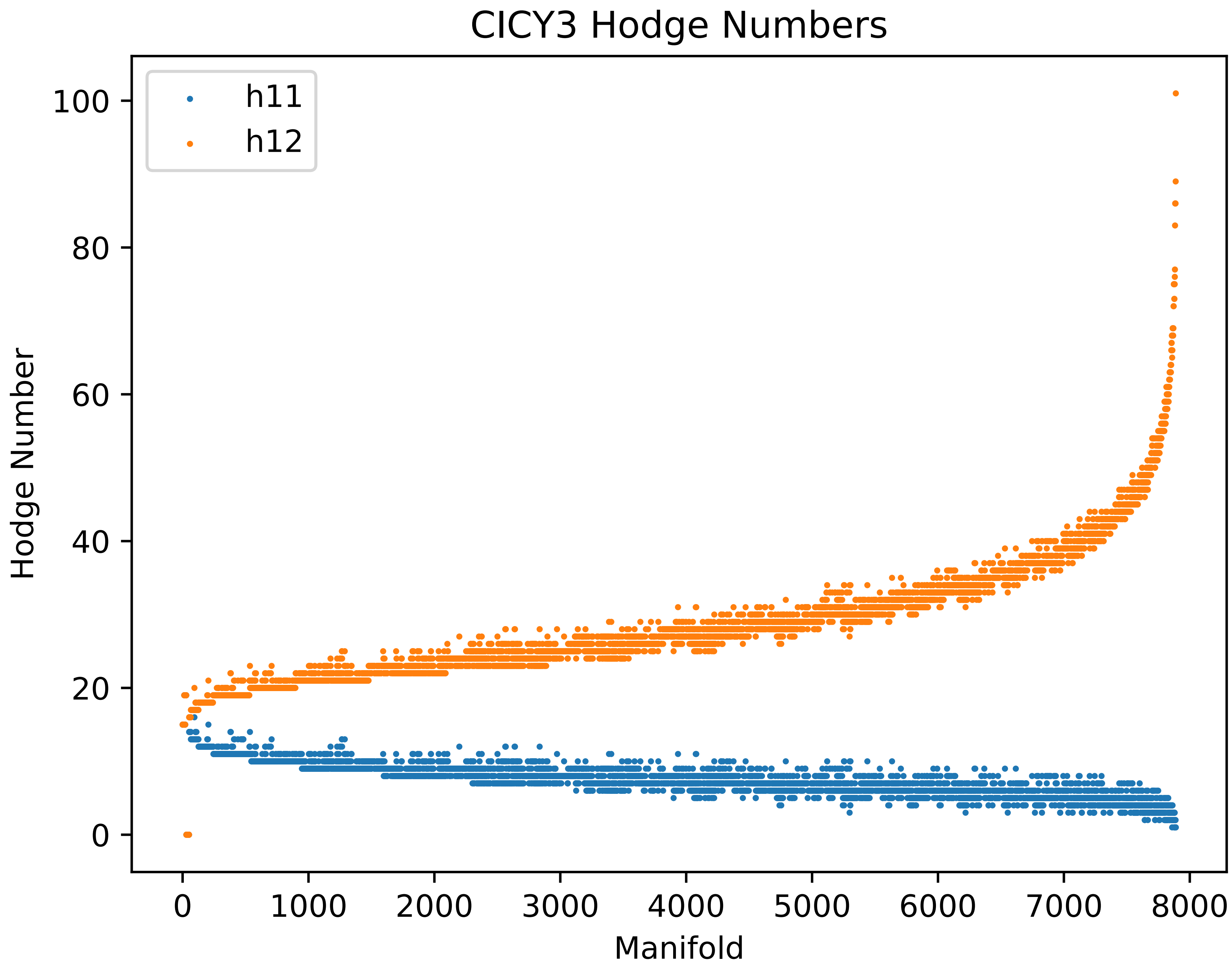
$$0 \leq h^{1,1} \leq 19 \ , \ \ 0 \leq h^{1,2} \leq 101$$

70 distinct Euler characters $\chi \in [-200, 0]$

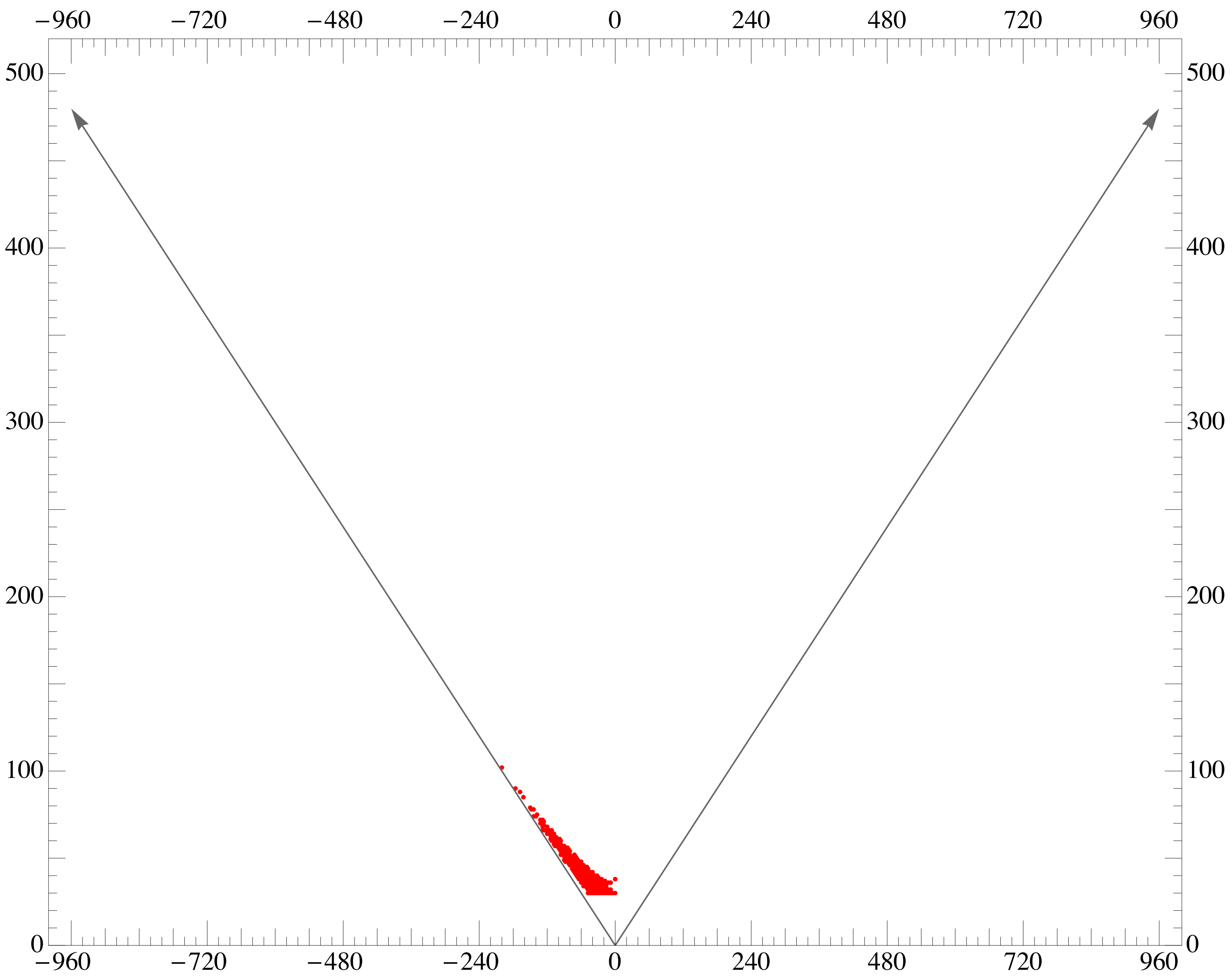
195 have freely acting symmetries, 37 different finite groups from $\mathbb{Z}_2$ to $\mathbb{Z}_8 \rtimes H_8$

Braun (2010)

CICY Hodge Numbers

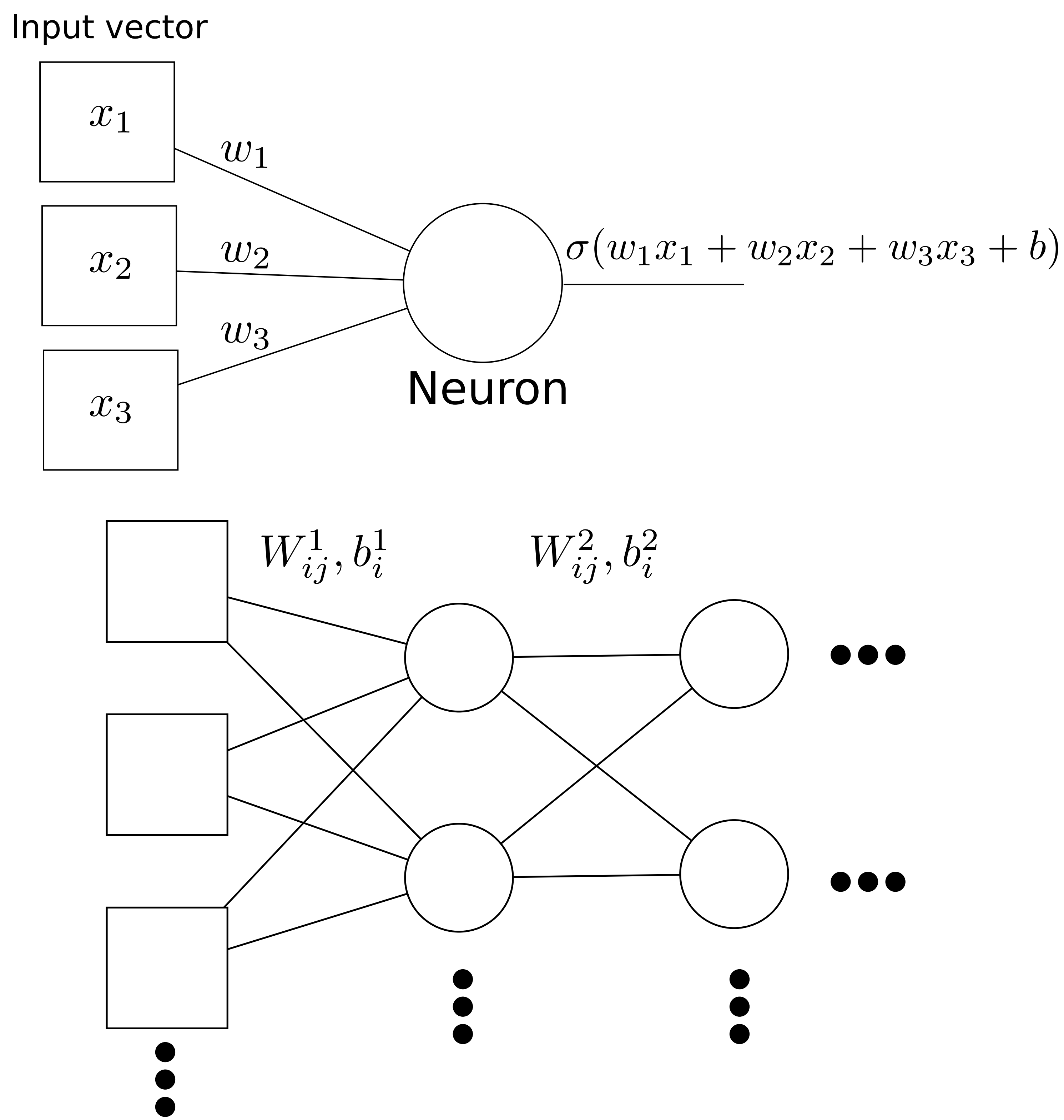


CICY Hodge Numbers



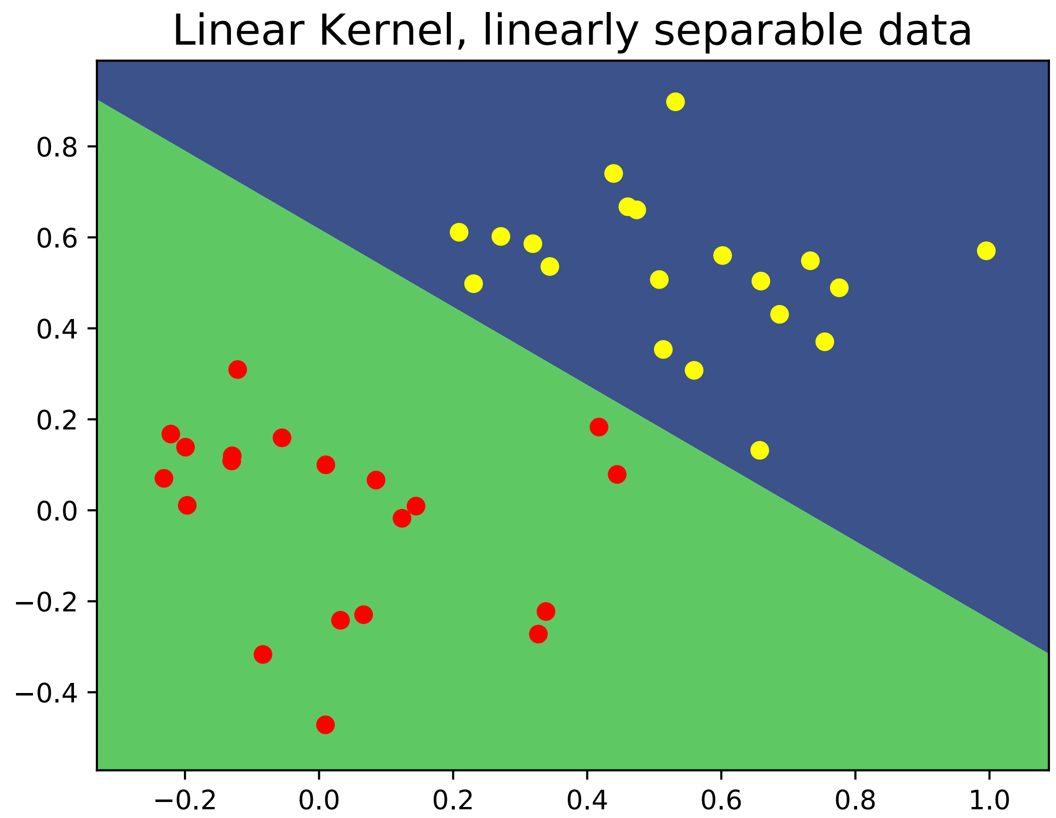
Candelas (2012)

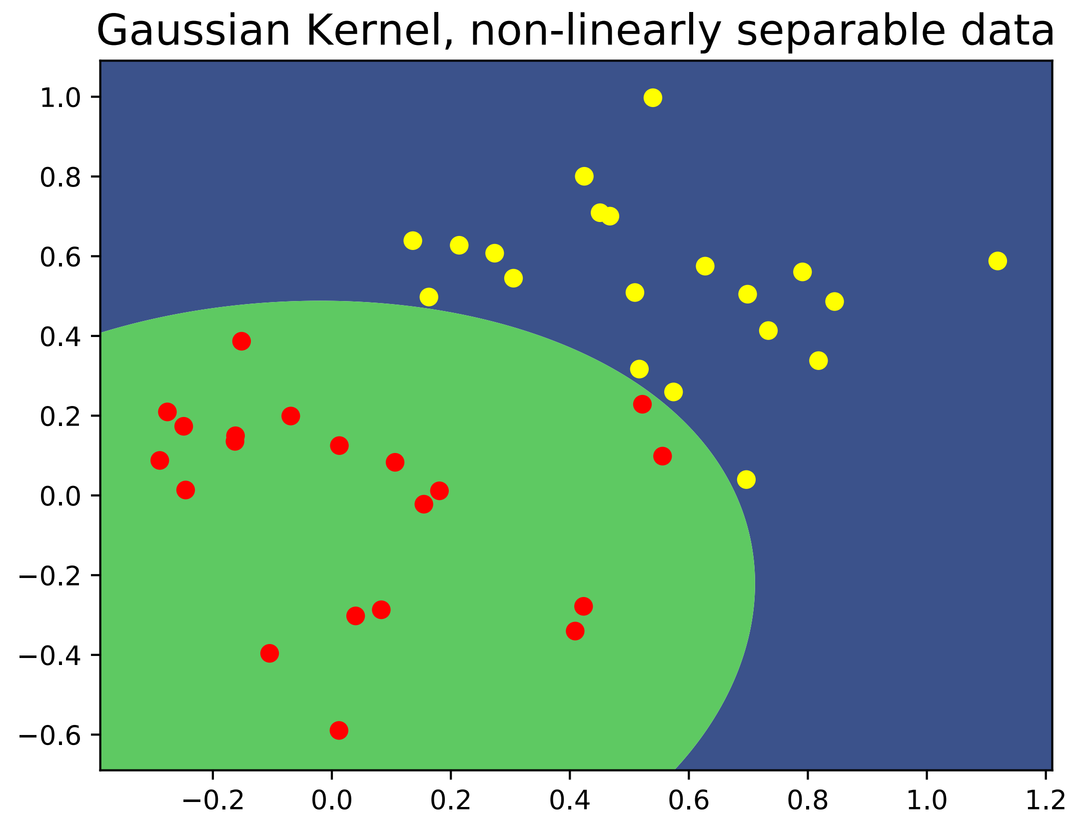
Feedforward Neural Networks



Schematic representation of feedforward neural network. The top figure denotes the perceptron (a single neuron), the bottom, the multiple neurons and multiple layers of the neural network.

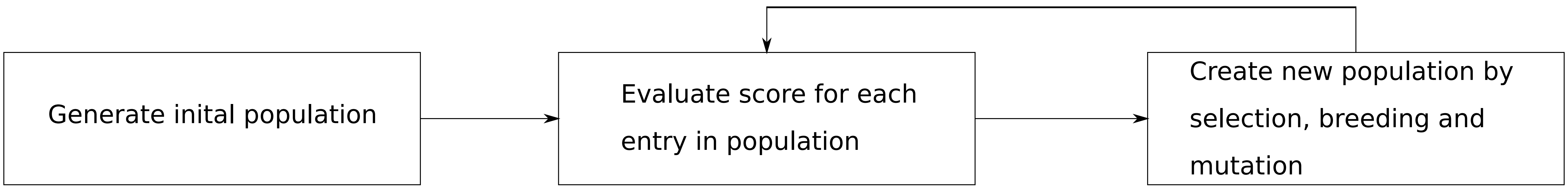
Support Vector Machines





SVM separation boundary calculated using our cvxopt implementation with a randomly generated data set.

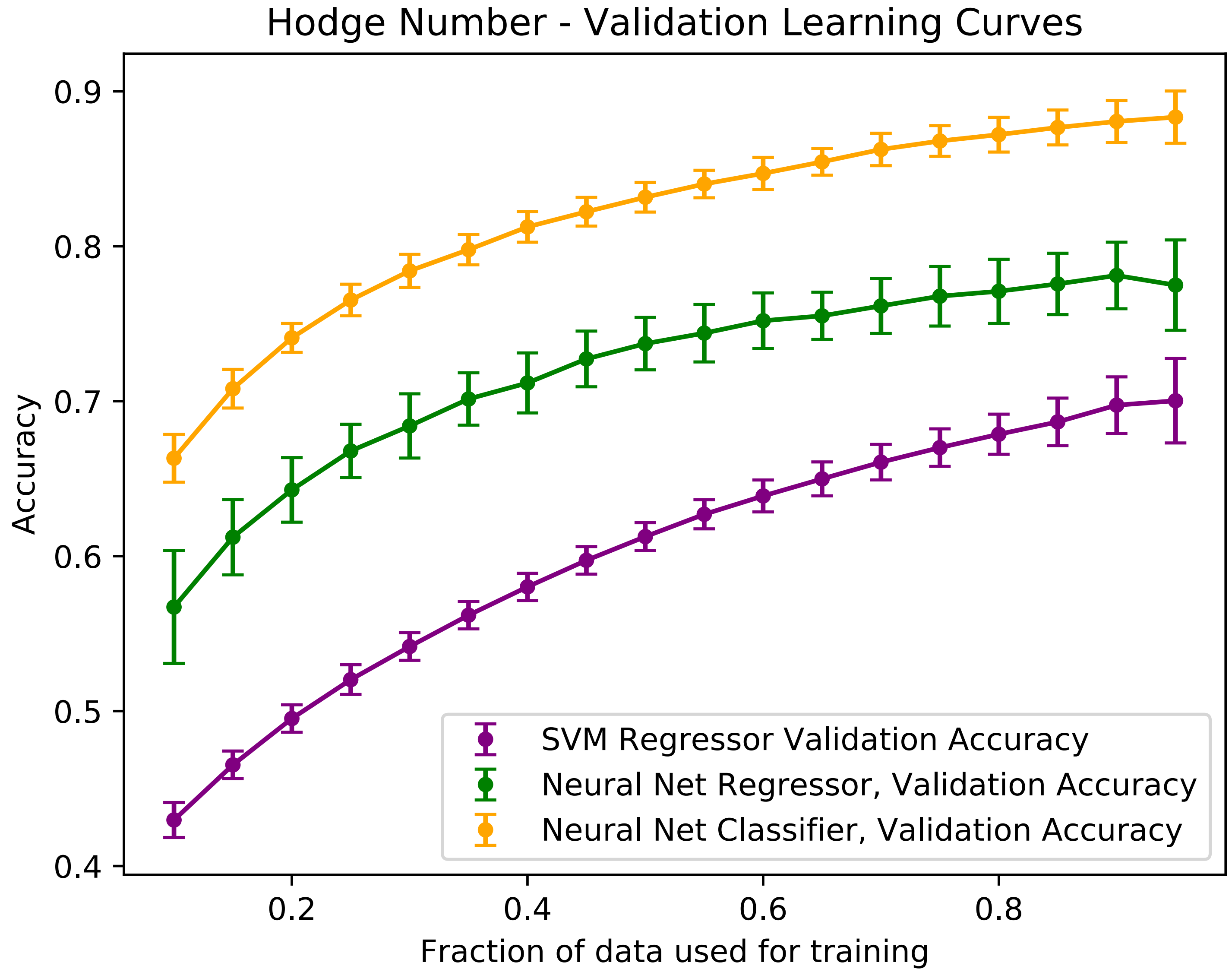
Genetic Algorithms



Used to fix hyperparameters (*e.g.*, number of hidden layers and neurons in them, activation functions, learning rates and dropout) in neural network.

Machine Learning h^{1,1}

- Since we know $\chi = 2(h^{1,1} - h^{1,2})$ from intersection matrix, we choose to machine learn $h^{1,1} \in [0, 19]$
- Previous efforts discriminated large and small $h^{1,1}$
- Use Neural Network classifier/regressor and SVM regressor



Bull, He, VJ, Mishra (2018)

Machine Learning $h^{1,1}$

	Accuracy	RMS	R^2	WLB	WUB
SVM Reg	0.70 ± 0.02	$\mathbf{0.53 \pm 0.06}$	$\mathbf{0.78 \pm 0.08}$	0.642	0.697
NN Reg	0.78 ± 0.02	0.46 ± 0.05	0.72 ± 0.06	0.742	0.791
NN Class	$\mathbf{0.88 \pm 0.02}$	-	-	**0.847**	**0.886**

$$\text{RMS} := \left(\frac{1}{N} \sum_{i=1}^{N} (y_i^{pred} - y_i)^2 \right)^{1/2} \qquad R^2 := 1 - \frac{\sum_i (y_i - y_i^{pred})^2}{\sum_i (y_i - \bar{y})^2}$$

$$\omega_{\pm} := \frac{p + \frac{z^2}{2n}}{1 + \frac{z^2}{n}} \pm \frac{z}{1 + \frac{z^2}{n}} \left(\frac{p(1-p)}{n} + \frac{z^2}{4n^2} \right)^{1/2}$$

Wilson upper/lower bounds
(WUB/WLB)

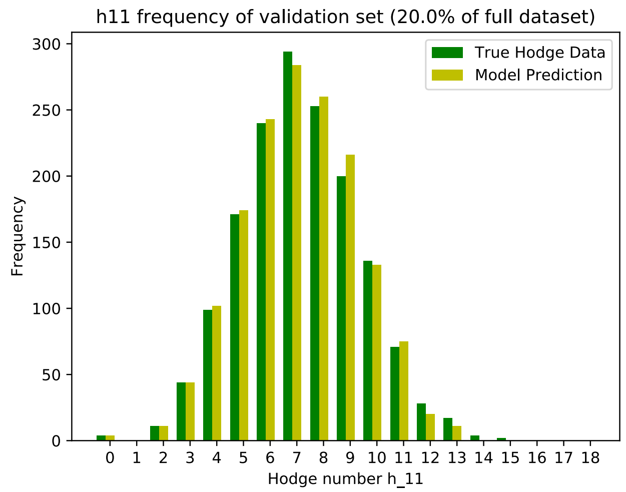
y_i	actual value
$\bar{y}$	average value
y_i^{pred}	predicted value
p	probability of successful prediction
z	probit
n	number of samples

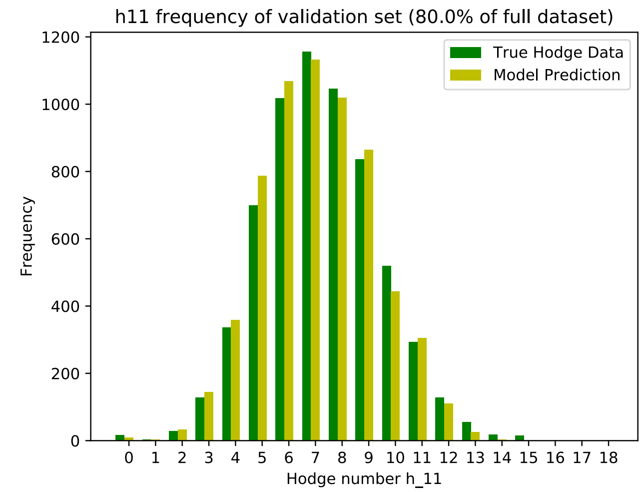
Bull, He, VJ, Mishra (2018)

Machine Learning $h^{1,1}$

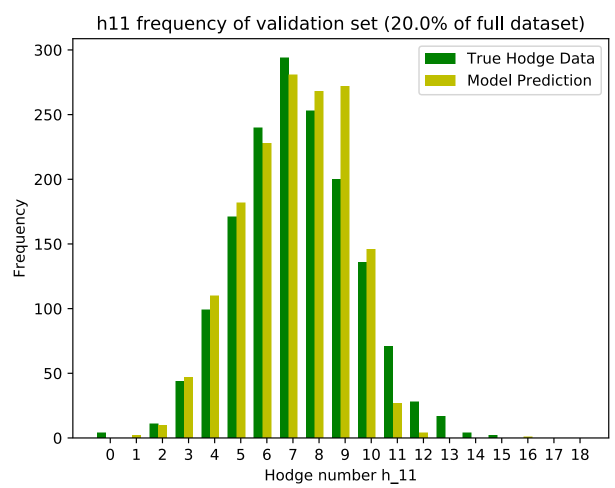
20% **80%**

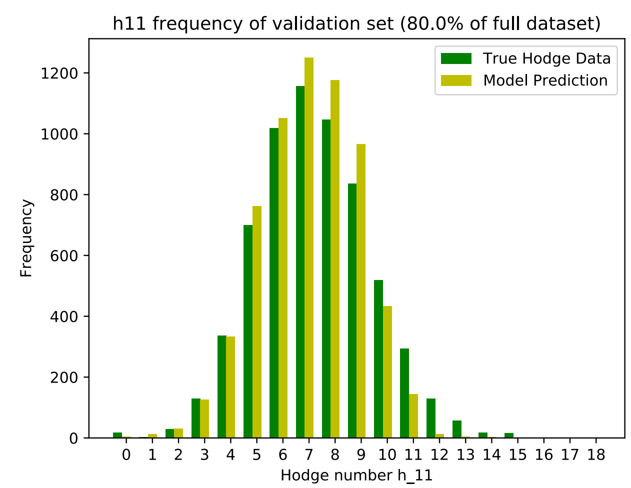
NN classifier



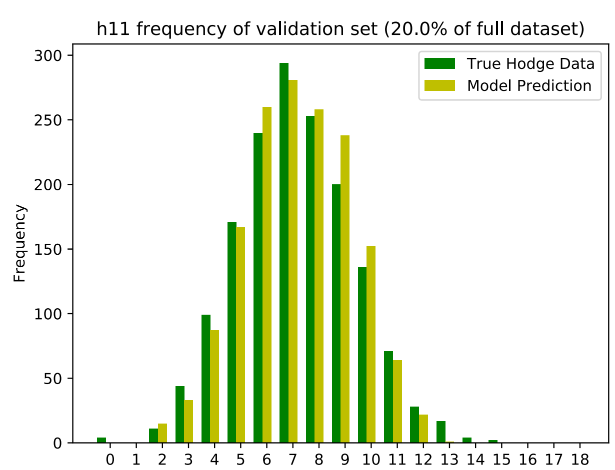


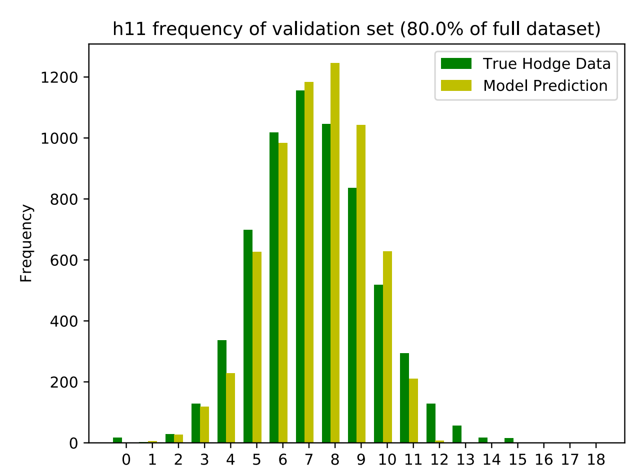
NN regressor





SVM regressor





Bull, He, VJ, Mishra (2018)

Favorability

- A CICY is **favorable** if its second cohomology class descends from that of the ambient space $\mathbb{A} = \mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_m}$

- *i.e.*, $h^{1,1} = \text{Number of } \mathbb{P}^n\text{s}$

- 4874 out of 7980 CICY configuration matrices are favorable

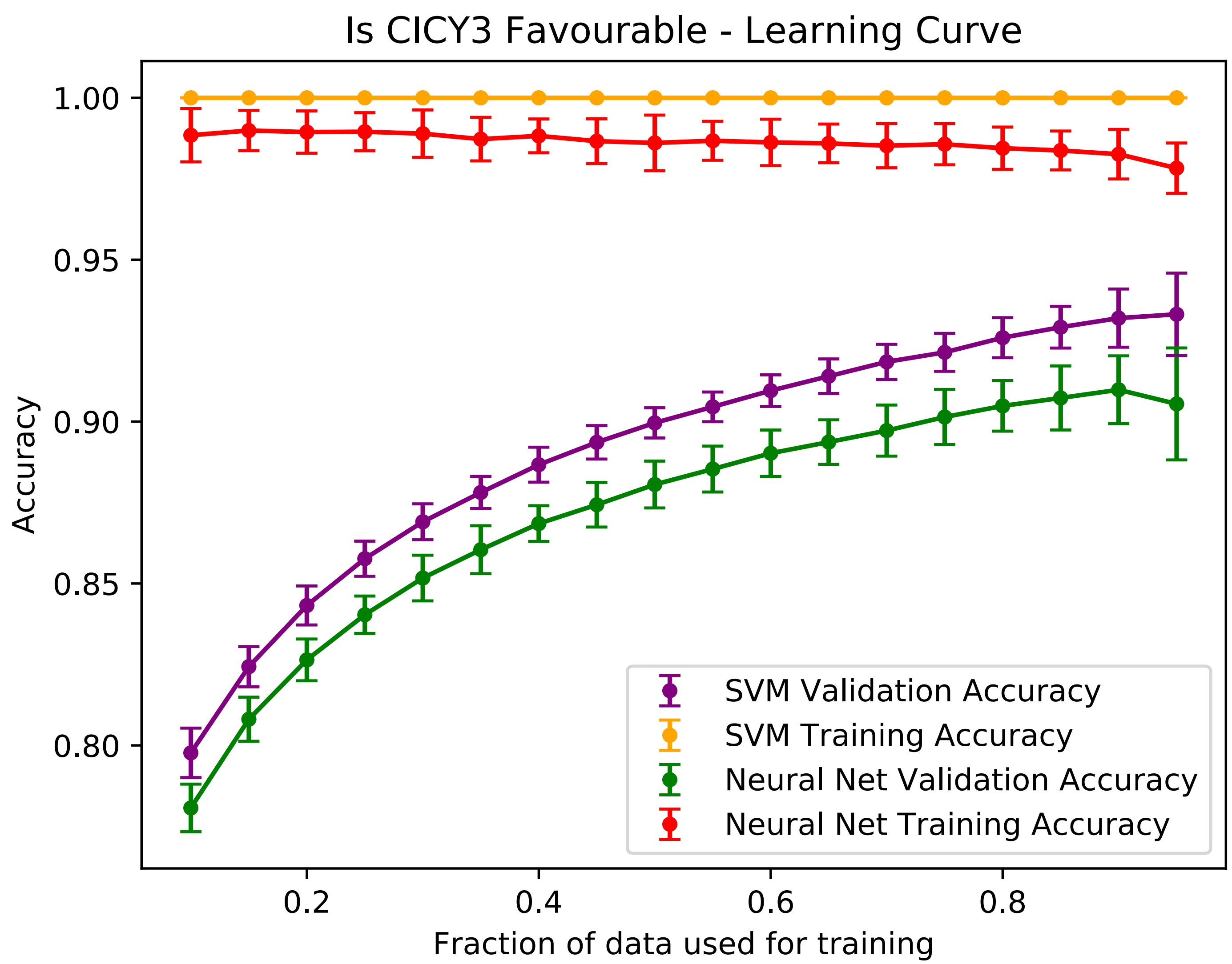
  this is 61%

- Favorable CICYs amenable to construction of stable vector bundles and monad bundles in string model building

- Use Neural Network and SVM to test whether manifolds are favorable

- This is a binary test

Favorability



	Accuracy	WLB	WUB
SVM Class	$\mathbf{0.933 \pm 0.013}$	0.867	0.893
NN Class	0.905 ± 0.017	**0.886**	**0.911**

Accurate and very fast!

Bull, He, VJ, Mishra (2018)

Discrete Symmetries

- Does a CICY enjoy a freely acting discrete symmetry?
- Useful for introducing Wilson lines to break GUT to Standard Model
- 195 of the CICYs have freely acting symmetries; 31 distinct groups, largest of order 32; 1695 CICY quotients possible
- 2.5% of the total dataset

Rare Results

- So far, we have used accuracy of results to benchmark success
- Suppose we are searching for a rare property within a dataset; perhaps it is there ~ 0.1% of the time
- A program that reports **NO** all the time is 99.9% accurate
- It is also completely useless!
- For *needle in a haystack* problems, accuracy is not a suitable benchmark
- Use SMOTE to increase rare entries by synthetically creating new ones

  (For our problem this turns out not to help a whole lot)

Better Metrics of Success

- Define **confusion matrix**

		Actual	
		True	False
Predicted Classification	True	True Positive (tp)	False Positive (fp)
	False	False Negative (fn)	True Negative (tn)

- From this, we construct

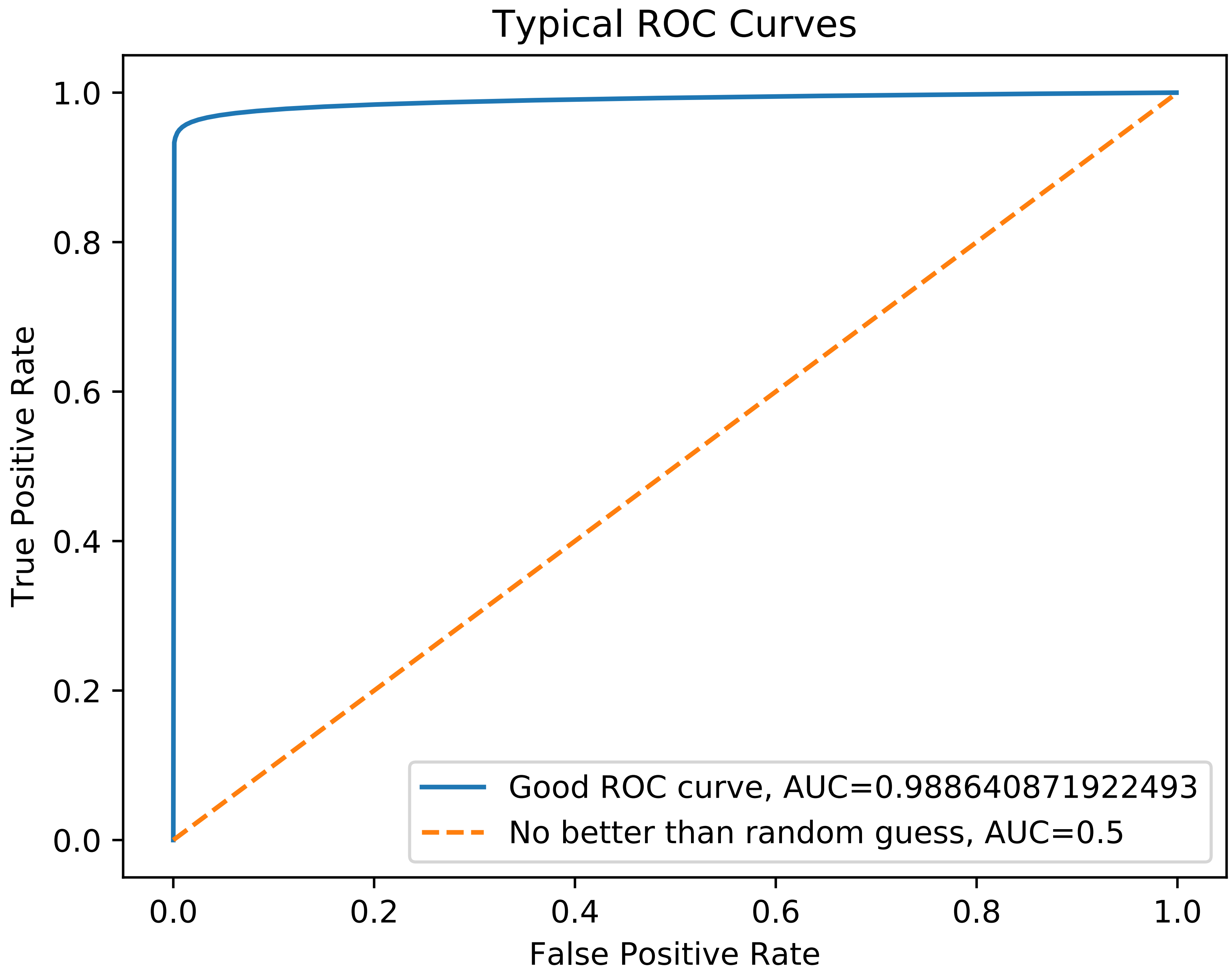
$$\text{TPR} := \frac{tp}{tp+fn} \qquad \text{FPR} := \frac{fp}{fp+tn}$$

$$\text{Accuracy} := \frac{tp+tn}{tp+tn+fp+fn} \qquad \text{Precision} := \frac{tp}{tp+fp}$$

- Then F-value is $F := \dfrac{2}{\frac{1}{\text{TPR}} + \frac{1}{\text{Precision}}} \in [0,1]$

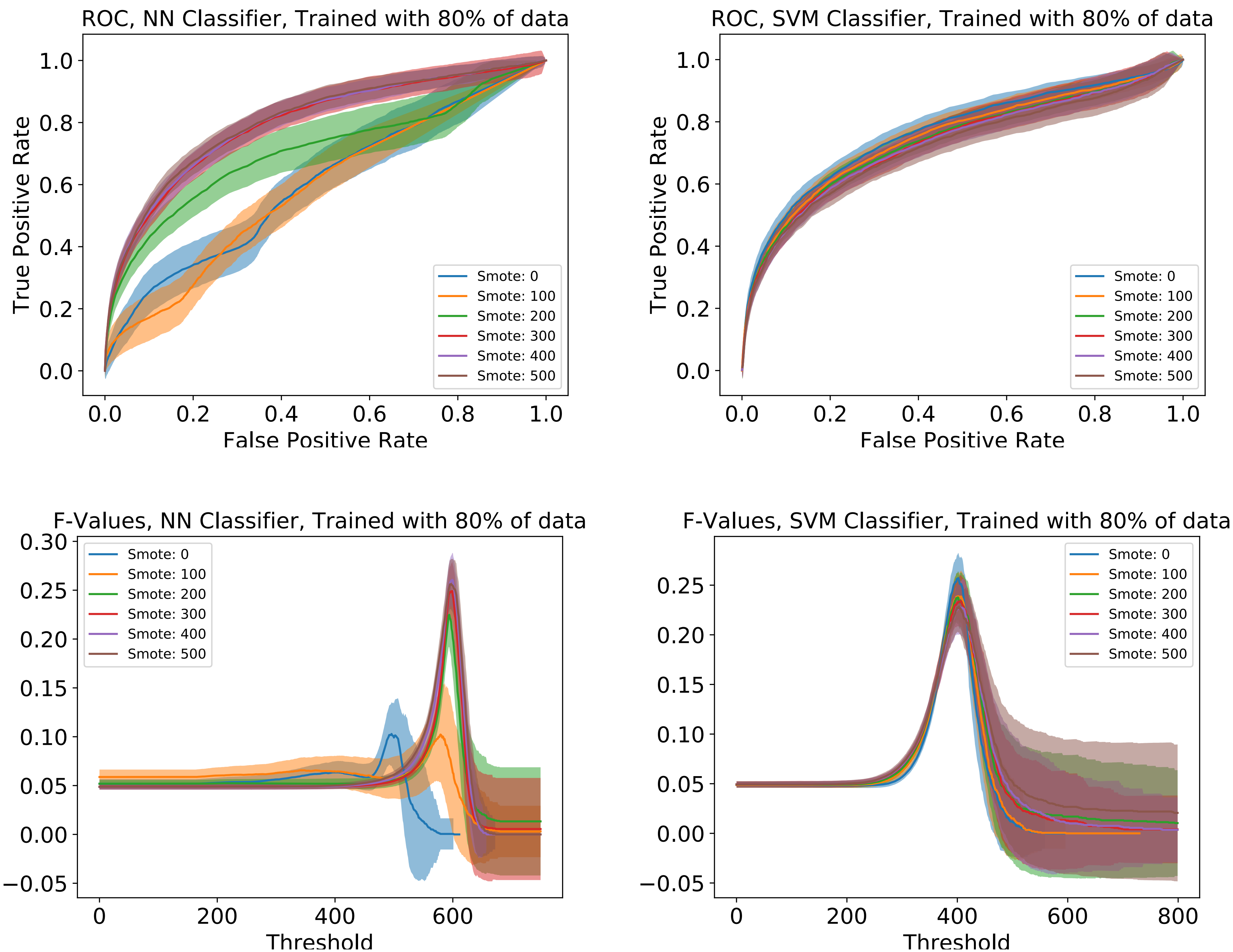
- Area Under R(eceiver) O(perating) C(haracteristic) Curve [AUC] plots TPR against FPR; this is between 0.5 and 1

ROC Curve



Typical ROC Curves
1.0
0.8
0.6
0.4
0.2
0.0
True Positive Rate
0.0
0.2
0.4
0.6
0.8
1.0
False Positive Rate
Good ROC curve, AUC=0.988640871922493
No better than random guess, AUC=0.5

ROC Curve & *F*-values



Bull, He, VJ, Mishra (2018)

ROC Curve & *F*-values

SMOTE	SVM AUC	SVM max F	NN AUC	NN max F
0	0.77 ± 0.03	0.26 ± 0.03	0.60 ± 0.05	0.10 ± 0.03
100	0.75 ± 0.03	0.24 ± 0.02	0.59 ± 0.04	0.10 ± 0.05
200	0.74 ± 0.03	0.24 ± 0.03	0.71 ± 0.05	0.22 ± 0.03
300	0.73 ± 0.04	0.23 ± 0.03	0.80 ± 0.03	0.25 ± 0.03
400	0.73 ± 0.03	0.23 ± 0.03	0.80 ± 0.03	0.26 ± 0.03
500	0.72 ± 0.04	0.23 ± 0.03	0.81 ± 0.03	0.26 ± 0.03

SMOTE 100 doubles the minority class, SMOTE 200 triples the minority class, etc.

SMOTE doesn't help SVM, helps Neural Network somewhat

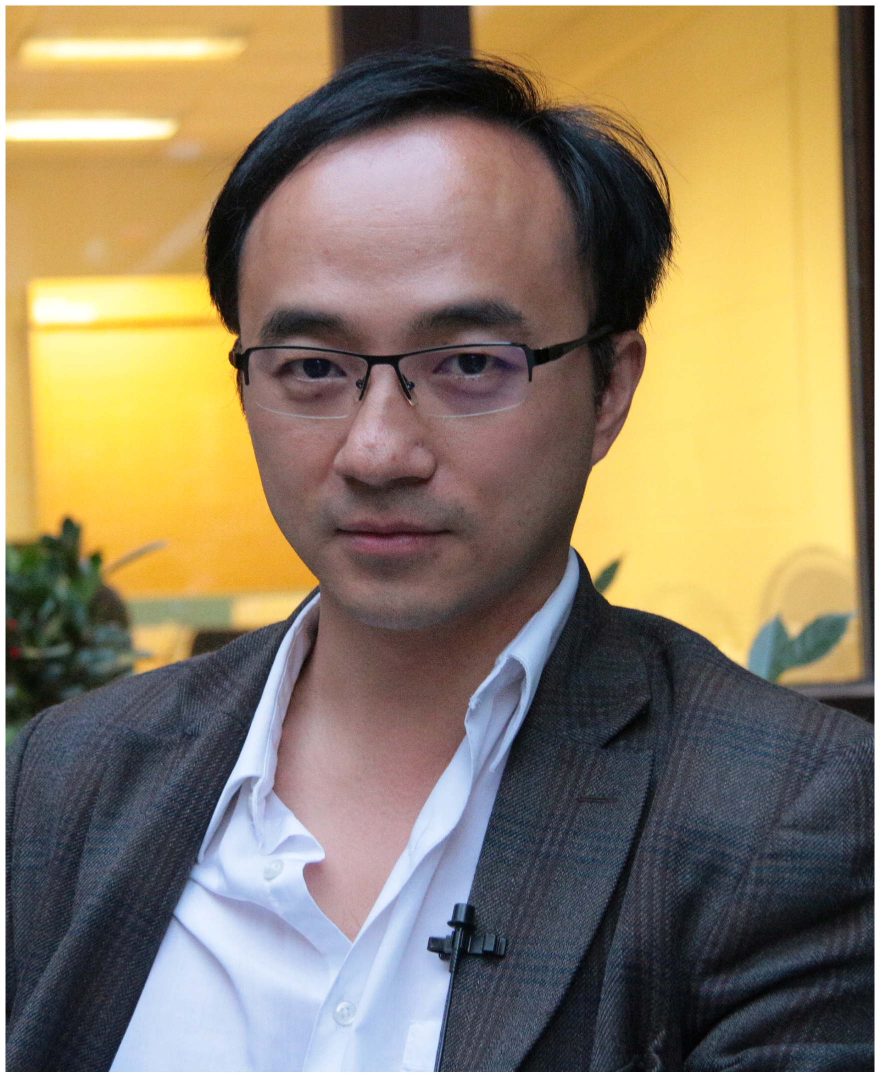
Shortlist 447 out of 1584 for further study; 417 of these are false positives; missed a quarter of manifolds with symmetries

Identifying geometries with discrete symmetries is a challenging problem

Bull, He, VJ, Mishra (2018)

Collaborators

Kieran Bull

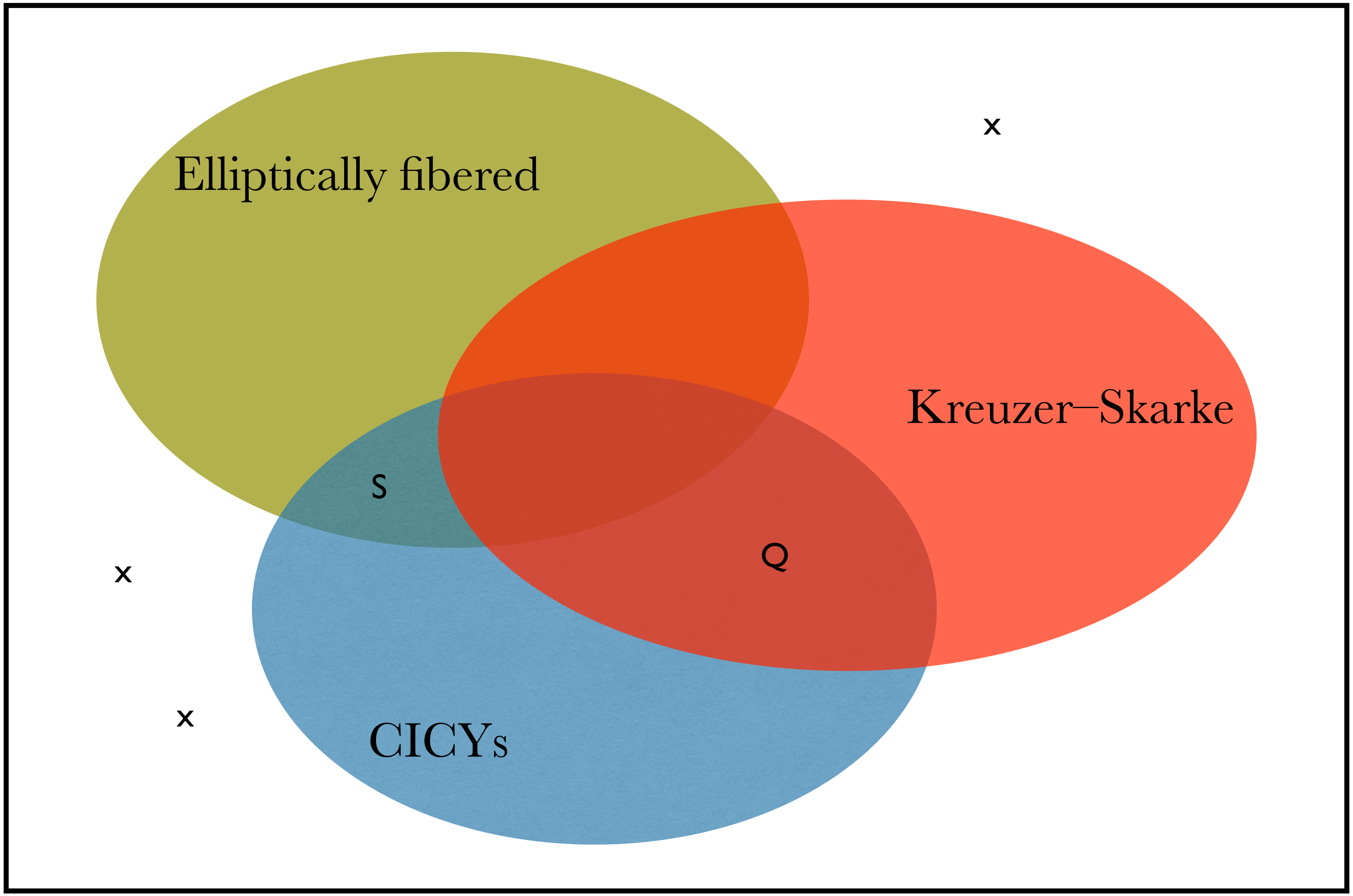


Yang-Hui He

Challenger Mishra

arXiv:1806.03121

Calabi–Yau Threefolds



Summary

- We have found new patterns in distribution of Hodge numbers in Kreuzer–Skarke dataset of reflexive polytopes
- We have applied machine learning to identify features of CICY threefolds
- In particular, we predict Hodge numbers with neural networks and support vector machines
- We test favorability as a property of CICY geometries
- We interrogate whether geometries have discrete symmetries
- Quick diagnostic tools for shortlisting geometries

Quo Vadis?

The Good

During the last 10-15 years, several international collaborations have computed geometrical and physical quantities and compiled them in vast databases that partially describe the string landscape

The Bad

Computations are hard, especially for a comprehensive treatment: dual cone algorithm (exponential), triangulation (exponential), Gröbner basis (double exponential), how to construct stable bundles over Calabi–Yau manifolds constructed from half a billion polytopes?

The Possibly Beautiful

Borrow techniques from “Big Data”

Prospectus

- Apply these ideas to study Kreuzer–Skarke dataset of reflexive polytopes and toric Calabi–Yau geometries constructed therefrom

- Extend analysis to CICY and toric fourfolds for F-theory model building

- Machine learn the Standard Model in string constructions

- Swiss cheese geometries for cosmological model building

- How does the black box learn semantics without syntax?

- Algebraic geometry and its intersection with physics is a wonderful landscape to explore with this new paradigm

Thank you!